Does Theology Matter? A Theoretical Look At Religious Terrorists and Their Organizations

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Abstract

If religious beliefs matter, how should we expect those beliefs to affect the behavior of religious terrorists? The religious motivation for terrorism often is assumed to be pursuit of benefits in the afterlife. Though Berman (2009) and others do not find much empirical support for theology as an motivator for terrorism, few economic models of terrorism attempt to incorporate religious beliefs. We explicitly model religious beliefs that may be held by terrorists, including expectations about the afterlife, blessings, punishment, or other supernatural responses to human activity. We consider whether religious beliefs might lead to behavior different than that of nonreligious terrorists, and whether the theoretical implications are consistent with the empirical evidence.

We employ an (after) life-cycle model of consumption and savings that allows accumulation of religious capital by sacrificing economic resources, and where religious capital yields expected rewards in the afterlife. We incorporate beliefs that suicidal attacks also might be rewarded in the afterlife. In this way, we consider the anticipation of reward and punishment in the afterlife as a motivation for sacrifice in the present world. The model is configured to reflect several common characteristics of religious terrorism, and optimal behavior is derived. The results suggest policy responses to lessen the appeal of terrorism and we determine the conditions under which such policies might prove effective.

We extend the analysis of the individual to that of the group. In particular, we consider the behavior of such agents within the club model developed by Iannoccone. We consider economic sacrifice as a signal of commitment to the group. Committed members might be entrusted with activities that are dangerous for the group, where the risk of defection otherwise is high.

The paper remains incomplete. Comments will be appreciated.
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1 Introduction

How should the economist characterize religious terrorists?\footnote{Thanks to my colleagues at Inforum, in the Economics Department at the University of Maryland, for their support of this project. Thanks also for the helpful comments offered by participants at the 2009 conference of the Association for the Study of Religion, Economics, and Culture at which an earlier draft of this paper was presented. This work remains incomplete.} As irrational fanatics whose unpredictable behavior defies comprehension? Certainly, the sacrifice of life by the suicidal terrorist defies the prediction of the standard consumption-labor model of optimal behavior, for often the cost seems far greater than the reward. Perhaps, though, this notion of reward is too narrow. If the terrorist believes that the sacrifice of his own life will bring great reward in the afterlife, then perhaps he is not so irrational, though his actions may be despicable. If, in addition, he values the welfare of his community, then he rationally might sacrifice his own worldly pleasures or even his life on behalf of others.

What can stop such religious terrorists from attacking others? Optimal policies to combat terrorism that does not consider of religious motivation may prove ineffective. To have any hope of weakening the inclination of religious terrorists to kill, we first must consider their motives and beliefs. While some motivations cannot be changed easily, such as obedience to perceived directives from God, the terrorist might rethink his suicidal commitment given suitable alternatives and conditions.

We thus construct an economic model of the individual terrorist who balances desires to gratify self in this world, the desire to serve God and community, and the desire for reward in the afterlife. The potential terrorist chooses between consumption, savings, and altruistic giving. At the same time, he chooses whether or not to conduct a suicidal attack. Both suicide and altruism may carry expected rewards in the afterlife and benefits in the natural world. Given these desires and beliefs, we derive optimal behavior from the perspective of the terrorist, and from there we consider policies that might reduce inclinations to destroy.

Recent research indicates that terrorists seldom act alone. Instead, most organize themselves in terrorist groups and networks. Religious terror groups seem especially deadly, and members of extreme religious groups seem especially devoted and loyal. We extend our analysis of the individual religious agent to consider groups of such agents. Members of the group consider whether to terrorize or to defect.

Though the belief seems common that suicidal terrorists seek reward in the afterlife and we begin with the premise that this understanding is correct, the notion is questioned by prominent scholars. These researchers report little evidence that such reward motivates terrorists, and alternative motives seem much more compelling. This paper does not counter those findings. Instead, we seek to understand better the implications of religious beliefs and how they align with observed behavior. The process may improve our understanding of the beliefs themselves, how belief motivates action, and whether policy might affect
those actions. We begin by constructing a model that incorporates the sort of religious beliefs that many understand to be common amongst religious terrorists. We then derive the logical and rational implications of such beliefs. Where the predicted behavior differs from the observed behavior of zealously religious terrorists, we turn back to the model as we consider why this might be so. In this way, we seek not only better understanding of religious radicals who commit atrocities, but we also develop a mechanism for considering broader questions about the activities of religious individuals. In particular, we consider why individuals who anticipate eternal consequences for present behavior demonstrate behavior that seems inconsistent with such beliefs, and where they seem to care too little about welfare in the afterlife and thereby sacrifice too little in the present world.

The following section presents a brief review of the literature on religion and economics. In particular, we review literature on religious altruism, suicide, and terrorism. Next, we describe a general model of savings, religious capital accumulation, and suicidal terrorism. We then solve a simplified version of the single-agent model for the two-period case and we analyze the results. Next, we consider the extension of the single-agent model to the multi-agent framework and the study groups of terrorists. Finally, we conclude.

2 The Literature


The first economic model of suicide was constructed by Hamermesh and Soss [8]. They model suicide of those unhappy with life, and they test their theory with U.S. data. Iannaccone [12] considers the motivations of religious martyrs. Stark, Doyle, and Rushing [22] and Watt [23] consider the impact of religion on suicide.
Krueger [13] offers statistical descriptions of terrorists. He reports on the economic and demographic characteristics of terrorists (see Table 2.4), though these particular findings are not specific to suicidal acts. He reports little evidence that income or education of the attacker drives terror, though terrorists more often attack wealthy countries. Attacks against occupying countries and by those in occupied countries are more common.

Berman [2] describes radical religious groups, both those that terrorize and those that do not. Terrorists seldom act alone, and so an understanding of the groups themselves is key. He considers what distinguishes religious terrorists from secular terrorist groups and what makes religious groups particularly effective. Berman notes that religious doctrine and commitment, military training, and tight-knit social fabric are not sufficient for success among religious terror groups. He argues that hateful ideology, thirst for revenge, and promise of reward in the afterlife do not seem particularly important as motivation for suicidal attacks, nor are suicide attacks typically executed by those who are depressed or otherwise suicidal. Not all groups that employ suicidal attacks are religious, and so promise of afterlife reward and other religious doctrine do not seem necessary. Anger and hate might be necessary conditions, but the relative rarity of attacks suggests that existence of these emotions are not sufficient to initiate terrorism. Though both secular and religious groups employ terrorism, religious groups are particularly deadly, even if religion is not a key motivator of individuals. Berman argues that the club model of religion fits the evidence particularly well. Though most terror groups fail, tightly knit radical religious communities are particularly effective in establishing a high defection constraints for their members, where members are particularly reluctant to be disloyal to the community. Radical religious groups typically establish systems of mutual aid and screen potential members to eliminate shirkers and others who would weaken the community; those sufficiently dedicated to be accepted by the group make prime candidates for activities that require even deeper commitment. This loyalty limits the success of attempts to counter terrorism, as it is difficult to bribe members and they are not easily enticed by promises of other personal gain. This loyalty to community seems a greater factor that religious doctrine. Indeed, many attackers seem to act on notions of altruism and heroism, where the sacrifice of own life is believed to provide significant, though perhaps exaggerated, benefit to the surviving community (page 11). The groups themselves often have political motives.

Berman and Laitin ([3], [4]) present club models of terrorism. In these models, agents signal commitment to the group through economic and other sacrifice. Potential free riders would be reluctant to make such sacrifices, and so they can be identified and excluded. Those who demonstrate commitment may be trusted to be loyal in cases where defection would destroy the group. Such loyalty among collaborators is essential to the successful mission of terror or conventional insurgency, where long periods of planning and preparation may allow ample opportunity for defection and carelessness. Berman and Laitin also argue that strong feelings of altruism toward the group and community may lead particular members to sacrifice their lives. In the first paper, they explain
why terrorists groups would sacrifice a valuable asset, the bomber. They argue that suicide attacks often are the only viable means for damaging a valuable but hardened target, and so suicide attacks are reserved for these cases. In their second paper, they again consider how religious terrorist groups can be unusually successful in deterring defection.

In Chapter 5, entitled "Can Suicide Bombers be Rational?", Wintrobe [24] presents the "solidarity multiplier." In this model, a member of a terrorist organization may be induced by some exogenous influence to desire greater solidarity with the group, thus deepening his involvement with the group and substituting group values and identity for his own. This may lead to the desire for still more solidarity, and hence a multiplier effect. In extreme cases, solidarity may dominate self interest to the point that suicide becomes rational. If members largely displace concern for self for the interests of the group, then this might explain how terror groups overcome the free rider problem, in which each member would prefer that another should actually execute the suicidal mission.


Brophy-Baermann and Conybeare [6] construct a model of terrorism and of optimal policy responses, and they test the model with data on terrorism in Israel. Lichbach [14] models inequality and the choice of rational agents to rebel. He finds that rational actors may choose to maximize pain in their opponents rather than to maximize their own pleasure.

3 The Single-Agent Model

Our objective is to model the potential religious terrorist who values his own economic condition, the condition of his community, and his state in the afterlife. While we acknowledge the importance of the terrorist organization for the efficacy of terrorism, we consider the individual terrorist. We do consider his concern for his community, but we do not model the terrorist cell or other community support that often is necessary to support such attacks. In addition to the sacrifice of life, the agent also may serve his community through economic sacrifice. These activities may be motivated both by personal preference and by religious beliefs. Religious motivations include anticipated reward through increased probability of entry to heaven and higher utility in the afterlife. We follow Blomberg, et al., in modeling religious giving as the accumulation of religious capital that may affect the level of utility in heaven, and we follow Pyne in modeling the effects of giving on destination probabilities.
3.1 The General Model

In this dynamic model, the agent enters each period with savings, a stock of religious capital, and, importantly, life. The agent receives an exogenous, perfectly-foreseen income stream. Given his preferences, his age, and the levels of the state variables, the agent decides how much to consume or save, how much to invest in religious capital, and whether to conduct an act of suicidal terrorism this period. The agent then carries out the consumption, savings, and investment decisions. If the agent decided to become a terrorist, this activity next is carried out and life in this world ends with certainty. If the agent decided against terrorism, then the agent may die at a known mortality rate. If the agent survives the current period, then he proceeds to the next and the process repeats. When the agent eventually dies due to nature or suicide he arrives immediately in one of three eternal states: non-existence, heaven, or hell. The agent holds subjective probability estimates for arrival at each destination, and these probabilities may depend on decisions made in the present world.

We allow the religious potential terrorist to have the following concerns for the present world:

\[
W(c_t, s_t, K) = W(u(c_t, s_t, K), U(s, K))
\]

where \(u\) is his own utility, \(U\) is the benefit to the community of the individual’s actions, \(c\) is the level of personal consumption, \(s\) is the stock of religious capital, and \(K\) is a the binary choice of whether to commit suicidal terrorism. For both own utility and social welfare, the net effect of killing may increase or lower welfare. Social welfare could be enhanced by supernatural reward following the suicide of a pious member, though we will focus on natural consequences. We assume concavity in \(c\) and \(s\) of \(W\) and its components, so that \(W_c > 0\) and \(W_s > 0\). Religious capital is accumulated as

\[
s_t = i_t + \delta s_{t-1}
\]

where \(i\) is current-period religious giving and \(\delta\) is the perceived survival rate for religious capital.

We consider the following perceptions of the afterlife:

\[
V(s_t, K) = (1 - p^N) \left[ p^B(K, s_t) V^B(K, s_t) + p^D(K, s_t) V^D(K, s_t) \right] + p^N V^N
\]

The perceived probability of nonexistence beyond this world is \(p^N\). Conditional on existence beyond this world, the individual anticipates that entry into heaven will be granted with probability \(p^B\) (probability of Bliss), and the probability of condemnation is \(p^D\) (probability of Destruction). Religious capital may affect these probabilities such that \(dp^B/ds \geq 0\) and \(dp^D/ds \leq 0\). The decision of suicide also may affect perceived destination probabilities and utility levels in the afterlife. The pleasure of life in heaven is \(V^B\), with \(V^B_s \geq 0\), and the misery of hell is \(V^D\), with \(V^D_s \geq 0\). In addition, we assume that second-order conditions ensure concavity for \(V^B\) and \(V^D\), and we assume that \(V^B > V^D\) for all \(s\) and \(K\). Note that \(p^B = 1 - p^D\), so that \(p^D_s = -p^B_s\) and \(p^D_{ss} = -p^B_{ss}\).
The derivative of afterlife utility with respect to religious capital is

\[
V_s (s, K_t) = (1 - p^N) \left[ p_s^B V^B + p^B V^B_s + p_s^D V^D + (1 - p^B) V^D_s \right] \\
= (1 - p^N) \left[ p_s^B (V^B - V^D) + p^B V^B_s + (1 - p^B) V^D_s \right]
\]

(1)

Religious capital thus acts both to increase the probability of gaining access to heaven and of avoiding condemnation and also to improve the expected utility across possible destinations. \( V \) increases with \( s \) so long as

\[
V^D \leq V^B + \frac{p^B V^B_s + (1 - p^B) V^D_s}{p_s^B}
\]

Since all terms on the right are non-negative and \( V^B > V^D \), the condition holds.\(^2\)

We assume that the potential terrorist of age \( \tau \) faces the following budget constraint, which requires expected solvency.

\[
A_t + \sum_{t=\tau}^{\min (\Upsilon, T)} R^{-(t-\tau)} \Phi_t (1 - K_{t-1}) (y_t - c_t - s_t + \delta s_{t-1}) \geq 0
\]

\( A \) is the level of temporal wealth accumulated by age \( \tau \) and \( R \) is the gross rate of return on temporal wealth. The income stream \( y \) is exogenous. The probability of survival to age \( t \) is \( \Phi_t \); this omits consideration of the suicide decision. This specification stands in contrast to Blomberg, et al., who omit the survival probability in the budget constraint. Instead, we assume that both borrowers and lenders have access to accurate actuarial tables and that lenders insist on expected solvency. We assume that lending is done by local bankers who anticipate terrorist activity that suicidal acts will be committed at age \( \Upsilon \), and these bankers require debt to be repaid before such activities can be carried out. Also in contrast to Blomberg, et al., is the inclusion in the budget constraint of a maximum temporal existence \( T \), rather than an infinite horizon specification. In fact, we do incorporate infinite horizons, but the budget constrain applies only to the temporal portion of existence. Otherwise, continued imposition of the budget constraint throughout eternity implies decaying religious capital in the afterlife, which stands in contrast to well-known religious teachings.

The potential terrorist chooses \( c \) and \( s \) for all \( t \in [\tau, T] \), in addition to choosing whether to commit suicide \( K \in \{0, 1\} \) at age \( \Upsilon \in [\tau, T] \), to maximize

\(^2\)\( V \) is concave in \( s \) if

\[
V_{ss} = (1 - p^N) \left[ p_s^B (V^B - V^D) + 2p^B_s (V^B_s - V^D_s) + p^B (V^B_s - V^D_s) + V^D_s \right] \leq 0
\]
Though it is slightly inconsistent with the notation, we specify for convenience $\Upsilon = 0$ as the decision not to kill, and we specify $K_{t<\Upsilon} = 0$ and $K_{\Upsilon} = 1$.

Conditional on survival to age $t$, a probability given by $P_t$, the probability of natural death at age $t$ is $P_t$. After rearranging, the first-order conditions for Equation 2 with respect to consumption and religious giving are

$$\beta^{t-\tau} \Phi_t W_c = \lambda R^{-((t-\tau)} \Phi_t$$

$$\beta^{t-\tau} \Phi_t [W_s + \max (K_t, P_t) V_s] = \lambda R^{-((t-\tau)} \Phi_t \left(1 - \frac{\delta (1 - P_t) (1 - K_t)}{R}\right)$$

Note that survival occurs with probability $(1 - P)(1 - K)$, and current-period capital investment is relevant in the following-period budget constraint only if the agent survives the current period. The optimal balance between consumption and giving is

$$W_s + \max (K_t, P_t) V_s = W_c \left(1 - \frac{\delta (1 - P_t) (1 - K_t)}{R}\right)$$

As the survival rate of religious capital increases and as the rate of return decreases, the marginal utility of religious capital relative to consumption must decrease. Thus, as less religious capital depreciates and as savings are rewarded more, consumption gives way to religious capital accumulation. As the probability of death increases, religious capital also is valued more. Given appropriate conditions, we can employ this equation to derive optimal consumption and giving paths.

With Equation 3, we can derive the intertemporal optimality condition for consumption as

$$W_{c,t-1} = \beta RW_c$$

In contrast to Blomberg, et al., the consumption path does not increase with survival rates because we incorporate mortality in the budget constraint. Any impact of mortality on the marginal utility of consumption will have a corresponding impact on the budget constraint. If the consumer anticipates looming death, then so too will prospective lenders anticipate the consumer’s pending
death. An increased desire to borrow will be met with unwillingness to lend, and thus the mortality terms disappear from the optimality condition.

We likewise derive the intertemporal optimality condition for religious capital.

\[
\frac{W_{s,t-1} + P_{t-1} \beta V_{s,t-1}}{R - \delta (1 - P_{t-1})} = \beta R \frac{W_{s,t} + \max (K_t, P_t) V_{s,t}}{R - \delta (1 - P_t) (1 - K_t)}
\] (6)

As the mortality rates increase with age, as patience increases, and as the rate of return on wealth increases, the path for religious capital accumulation becomes steeper. Optimal giving rules depend on the suicide decision, but for the condition to be relevant, the agent must decide against suicide at time \( t - 1 \). In contrast to the consumption condition, religious capital accumulation depends directly on mortality and on beliefs about the afterlife. This is so because we assume that consumption decisions do not affect \( V \). If instead we allowed consumption decisions (such as asceticism or gluttony) to affect afterlife conditions, then the optimality conditions for consumption and religious capital would be similar.

The potential terrorist will kill in period \( \tau \) if \( \mathcal{L}^\tau (s_\tau, A_\tau) > \mathcal{L}^t (s_\tau, A_\tau) \), for all periods \( t \), and \( \mathcal{L}^0 (s_\tau, A_\tau) > \mathcal{L}^\tau (s_\tau, A_\tau) \), where \( \mathcal{L}^t \) denotes utility given the decision to kill in period \( t \) and where \( \mathcal{L}^0 \) indicates discounted utility given the decision not to kill. To rule out the possibility that the suicidal individual will borrow heavily to fund high consumption before certain death, we assume that lending is done by local bankers who anticipate terroristic activity, and these bankers require debt to be repaid before such activities can be carried out.

We thus have derived the optimal balance between spending on consumption and religious capital, and we derived the preferred time paths for both. When these are combined with the budget constraint and initial conditions, we can derive optimal levels for consumption, savings, and religious capital accumulation, and we characterize the terrorism decision.

### 3.2 Quadratic Utility

To simplify further derivations, we specify polynomial equations for the model. We assume that monotonicity, concavity, and other assumptions hold for relevant levels of consumption and religious capital, and later we will derive the required regularity conditions. We further simplify the model to make analytical derivations tractable, and we focus on a set of beliefs commonly understood to be held by many terrorists. After presenting the model in the general case, we simplify the \( T \)-period model, and finally we fully solve a the two-period case.

The quadratic current-period utility function is

\[
W (c_t, s_t, K) = \theta c_t - \frac{\Theta}{2} c_t^2 + \gamma s_t - \frac{\Gamma}{2} s_t^2 + \sigma K_t
\]

where the parameters on consumption and capital are positive. The parameter on terrorism either could be positive or negative. Recall that \( W \) denotes social utility from the perspective of the individual, so that parameters on capital and
terrorism reflect both personal preferences and impacts on the surrounding community. For this reason, the individual’s preference for life might be outweighed by benefits to the community from a terroristic act, such that the net effect is \( \sigma > 0 \). By community, we mean the terrorist’s family or religious group, but perhaps not the victims of the attack. Still, if revenge motivates the terrorist, then he might derive satisfaction from the pain of his enemy, and this too would yield \( \sigma > 0 \).

We assume that earthly perceptions of utility in heaven is finite, takes a quadratic form in religious capital, and is linear in the (binary) choice to kill. While the parameter on killing again could take either sign, we will focus on expected rewards for killing. We assume that utility in hell does not depend on capital or terrorism.

\[
V_B(s_t, K_t) = B + \omega s_t - \frac{\Omega}{2} s_t^2 + \Sigma K_t
\]

\[
V_D(s_t, K_t) = -D
\]

We maintain that \( V_B > V_D \) and that \( V_s^B \geq 0 \) for all relevant levels of \( s \) and \( K \).

Finally, we specify the perceived probability of admittance to heaven as

\[
p_B(K_t) = \pi + \Pi K_t
\]

where \( 0 \leq p_B \leq 1 \). Parameter \( \pi \) denotes the probability of admittance barring any qualifying activity other than adoption of faith. For simplicity, we do not consider the belief that religious capital affects this probability. Instead, the probability may be affected only by the decision to kill. Again, admittance is granted according to faith alone and not according to works, except for martyrdom. We further assume that \( \Pi \equiv 1 - \pi \), so that admittance to heaven is assured if the agent dies while committing a terroristic act. This implies

\[
p_B(K_t) = \begin{cases} 
\pi & : K = 0 \\
1 & : K = 1
\end{cases}
\]

For simplicity, we assume that the agent’s faith is sufficient so that the perceived probability of non-existence after death is zero (\( p_N = 0 \)). This assumption is strong. Recall that the probability of condemnation to hell is \( p_D = 1 - p_B(K) \).

Given these assumptions, we calculate the following derivatives. The marginal utility of consumption is \( W_c = \theta - \Theta c \), of religious capital in the natural life is \( W_s = \gamma - \Gamma s \), religious capital in the afterlife is \( V_s^B = \omega - \Omega s \), and the impacts of religious capital on utility in hell and on entry probabilities are \( V_s^D = p_s^B = 0 \).

The optimal time path for consumption, corresponding to Equation 5, is

\[
c_{t-1} = \beta R c_t + (1 - \beta R) \frac{\theta}{\Theta} \tag{7}
\]

To derive the optimal accumulation of religious capital, we first must derive the marginal utility of capital in the afterlife. Building on Equation 1 with the given specifications for utility and probability functions, we find
\[ V_s(s, K) = p^B_s (V^B - V^D) + p^B_s (V^B - V^D_s) + V^D_s \]

\[ = (\pi + (1 - \pi) K_t) (\omega - \Omega s_t) \]

\[ = (K_t + (1 - K_t) \pi) (\omega - \Omega s_t) \]

Given the decision of whether to kill, we find that \( V_s \) is an affine transformation of \( s \). In the case of natural death, the function is

\[ V_s(s, 0) = \pi \omega - \pi \Omega s_t \]

and in the case of suicidal terrorism, the derivative is

\[ V_s(s, 1) = \omega - \Omega s_t. \]

If entry to heaven was guaranteed unconditionally, then there would be no distinction between the derivatives, though a difference in utility levels would remain. The intertemporal optimality condition for accumulating religious capital, corresponding to Equation 6, is

\[ \frac{\gamma - \Gamma s_{t-1} + P_{t-1} \pi (\omega - \pi \Omega s_{t-1})}{R - \delta (1 - P_{t-1})} = \beta R^\gamma \frac{\gamma - \Gamma s_{t} + (K_t + (1 - K_t) \pi P_t) (\omega - \Omega s_t)}{R - \delta (1 - P_t) (1 - K_t)} \]

For \( K_t = 0 \), we derive

\[ s_{t-1} = \left[ \frac{\gamma + P_{t-1} \pi \omega}{\Gamma + P_{t-1} \pi \Omega} - \beta R \frac{(R - \delta (1 - P_{t-1})) \gamma + \pi P_t \omega}{R - \delta (1 - P_t) \Gamma + P_{t-1} \pi \Omega} \right] \]

\[ + \beta R \frac{R - \delta (1 - P_{t-1})}{R - \delta (1 - P_t) \Gamma + P_{t-1} \pi \Omega} \frac{1}{s_t} \]

and for \( K_t = 1 \), we find

\[ s_{t-1} = \left[ \frac{\gamma + P_{t-1} \pi \omega}{\Gamma + P_{t-1} \pi \Omega} - \beta R \frac{(R - \delta (1 - P_{t-1})) (\gamma + \omega)}{(\Gamma + P_{t-1} \pi \Omega) (\Gamma + \Omega) s_t} \right] \]

Finally, corresponding to Equation 4, the optimal balance between consumption and religious-capital accumulation is

\[ (\gamma - \Gamma s_t) + (K_t + (1 - K_t) \pi P_t) (\omega - \Omega s_t) = (\theta - \Theta c_t) \left( 1 - \frac{\delta (1 - P_t) (1 - K_t)}{R} \right) \]

or

\[ s_t = \frac{\gamma - \theta [1 - \delta/R (1 - P_t) (1 - K_t)] + (K_t + (1 - K_t) \pi P_t) \omega}{\Gamma + (K_t + (1 - K_t) \pi P_t) \Omega} \]

\[ + \Theta [1 - \delta/R (1 - P_t) (1 - K_t)] (1 - \pi) \]

\[ \frac{1}{\Gamma + (K_t + (1 - K_t) \pi P_t) \Omega} \]

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3.3 A Simplified Quadratic Model

We now turn to a $T$-period case, where we have in mind $T$ stages of life, ultimately ending in death, in which suicidal terrorism is feasible. We consider $T + 1$ relevant cases: no suicide and suicide in period $Y \in \{1, T\}$. Note that death is assured at the end of stage $T$ ($P_T = 1$). We assume initial conditions of zero wealth and zero religious capital. We make the simplifying assumptions that $\delta = R = \beta = 1$, that $\Omega = 0$, and we impose the normalization $\Gamma = 1$.

3.3.1 Case: No Killing

We consider the case of no killing, where $K_t = 0 \forall t \in \{1, T\}$. The optimal level of consumption for the case of no killing is constant

$$c_{t-1} = c_t$$

and so we can write consumption in terms of its level in the final potential period of natural life: $c_t = c_T$.

The optimal religious capital accumulation path, assuming that $P_t > 0$, may be written

$$s_{t-1} = \left[1 - \frac{P_{t-1}}{P_t}\right] \gamma + \frac{P_{t-1}}{P_t} s_t$$

By iterating on this equation and then simplifying, we solve for investment in $t$ as a function of anticipated investment in the final period, recalling that $P_T = 1$:

$$s_t = \left[1 - \frac{P_t}{P_T}\right] \gamma + \frac{P_t}{P_T} s_T$$

$$= (1 - P_t) \gamma + P_t s_T$$

By combining the optimal investment-consumption ratio and the optimal consumption path, we find capital in terms of the optimal consumption level:

$$s_t = \gamma + P_t [\gamma c_t - \theta + \pi \omega + \Theta c_t]$$

Recalling that consumption is expected to sustain a constant level, this result indicates that the optimal capital stock varies over time only to the extent that mortality rates vary. Surely this result is a consequence of our strong assumptions. Still, it implies that if mortality rates increase with age, then so will capital stock rise as death approaches. The result also suggests that the agent may borrow while young to build up stock in religious capital just in case he dies an early natural death. For later use, note that if we substitute the equation for $s_T$ in terms of $c_T$ we can calculate

$$s_t^2 = \left[\gamma - P_t \theta + P_t \pi \omega + P_t \Theta c_T\right]^2$$

$$= \left[\gamma + 2P_t \gamma \theta + 2P_t \gamma \pi \omega + P_t \pi \omega \theta - 2P_t \pi \omega \theta + P_t \pi \omega \pi \omega\right] + 2P_t \Theta \left[\gamma - P_t \theta + P_t \pi \omega\right] c_T + P_t P_t \Theta \Theta c_T c_T
and
\[ s_t - s_{t-1} = [\gamma - P_t \theta + P_t \pi \omega + P_t \Theta c_T] - [\gamma - P_{t-1} \theta + P_{t-1} \pi \omega + P_{t-1} \Theta c_T] \]
\[ = (P_t - P_{t-1}) [-\theta + \pi \omega + \Theta c_T] \]

In the case of natural death, the derivative with respect to religious capital of afterlife utility in the case of no suicide is
\[ V_s(s_t, 0) = \pi \omega \]

The lifetime budget constraint is
\[ \sum_{t=1}^{T} \Phi_t [c_t + s_t - s_{t-1}] \leq \sum_{t=1}^{T} \Phi_t y_t \]

We now have derived \( c_t \) and \( s_t \) as functions of \( c_T \). We can employ the budget constraint to find the optimal \( c_T \) from the perspective of the first period. Since the unconditional probability of death in period \( t \) is \( \Phi_t P_t \) and death by age \( T \) is certain, then \( \sum_{t=1}^{T} \Phi_t P_t = 1 \). Recall that \( p_T = 1 \) and assume \( p_0 = 0 \). By employing in the budget constraint the equation for optimal investment in terms of consumption, we find
\[ \sum_{t=1}^{T} \Phi_t y_t \leq \sum_{t=1}^{T} \Phi_t [c_t + s_t - s_{t-1}] \]
\[ = \sum_{t=1}^{T} \Phi_t [c_T + [\gamma - P_t \theta + P_t \pi \omega + P_t \Theta c_T] - [\gamma - P_{t-1} \theta + P_{t-1} \pi \omega + P_{t-1} \Theta c_T]] \]
\[ = \gamma + \sum_{t=1}^{T} \Phi_t [c_T + [\gamma - P_t \theta + P_t \pi \omega + P_t \Theta c_T] - [\gamma - P_{t-1} \theta + P_{t-1} \pi \omega + P_{t-1} \Theta c_T]] \]
\[ = c_T \sum_{t=1}^{T} \Phi_t [1 + P_t \Theta - P_{t-1} \Theta] + \gamma \sum_{t=1}^{T} \Phi_t [1 + (P_t - P_{t-1}) \Theta] \]

which implies that the optimal level of consumption for all periods, conditional on no suicide (denoted by superscript 0), is
\[ c^0 = \frac{\sum_{t=1}^{T} \Phi_t y_t - \gamma - (\theta + \pi \omega) \sum_{t=1}^{T} \Phi_t (P_t - P_{t-1})}{\sum_{t=1}^{T} \Phi_t [1 + (P_t - P_{t-1}) \Theta]} \] (8)

We write the lifetime utility equation, given the decisions against killing, as
\[ L_0 (s_1, A_1) = \sum_{t=1}^{T} \Phi_t \left[ \theta c_t - \frac{\Theta}{2} c_t^2 + \gamma s_t - \frac{1}{2} s_t^2 \right] + P_t [\pi (B + \omega s_t) - (1 - \pi) D] \]

After employing in equations for consumption and investment in terms of \( c^0 \), we find utility as a function of consumption (note again that \( \sum_{t=1}^{T} \Phi_t P_t = 1 \)):

\[ L_0 (s_1, A_1) = \sum_{t=1}^{T} \Phi_t \left[ \theta c_t - \frac{\Theta}{2} c_t^2 + \gamma s_t - \frac{1}{2} s_t^2 + P_t [\pi (B + \omega s_t) - (1 - \pi) D] \right] \]

so that

\[ L_0 (s_1, A_1) = \sum_{t=1}^{T} \Phi_t \left[ \theta c^0 - \frac{\Theta}{2} c^0 c^0 + (\gamma + P_t \pi \omega) s_T - \frac{1}{2} (s_T)^2 + P_t [\pi B - (1 - \pi) D] \right] \]

where \( c^0 \) is optimal consumption and where "0" denotes the decision not to kill in any period. If we expanded \( c^0 \), we would find an equation for lifetime expected utility as a quadratic function of lifetime expected income, perceptions of the afterlife, and preferences.

### 3.3.2 Case: Killing in Period \( \Upsilon \)

We now consider the case of killing in period \( \Upsilon \), where \( K_\Upsilon = 1 \) for \( \Upsilon \in \{1, T\} \). In the case of suicidal terrorism committed in stage \( \Upsilon \) of life, the derivative of afterlife utility with respect to the stock of religious capital is

\[ V_s (s, K) = \omega \]

If natural death occurs before the planned suicide can take place, then

\[ V_s (s_t, 0) = \pi \omega \]

Optimal consumption again is constant, so we can specify the path in terms of the chosen final period \( \tau \):

\[ c_t = c_\Upsilon \tag{9} \]

We found earlier the optimal religious-capital accumulation path, for \( K_{\tau<\Upsilon} = 0 \),

\[ s_{t-1} = \left[ 1 - \frac{P_{t-1}}{P_t} \right] \gamma + \frac{P_{t-1}}{P_t} s_t \]

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For $K_T = 1$, we find

$$s_{T-1} = (1 - P_{T-1}) \gamma - P_{T-1} (1 - \pi) \omega + P_{T-1} s_T$$

By iterating backwards with these two equations, we find the optimal capital stock in terms of final-period stock for $t < \tau$

$$s_t = [1 - P_t] \gamma - P_t (1 - \pi) \omega + P_t s_T$$

In the chosen final period $T$, the optimal balance between consumption and investment in religious capital is

$$s_T = (\gamma - \theta + \omega) + \Theta c_T$$

Otherwise, for $t < T$,

$$s_t = \gamma - P_t \theta + P_t \pi \omega + P_t \Theta c_t$$

$$s_T - s_t = (\gamma - \theta + \omega) + \Theta c_T - [\gamma - P_t \theta + P_t \pi \omega + P_t \Theta c_T]$$

$$= - (1 - P_t) \theta + (1 - P_t \pi) \omega + (1 - P_t) \Theta c_T$$

We see that optimal level of capital increases in the period chosen for martyrdom by

$$s_T - s_t = (\gamma - \theta + \omega) + \Theta c_T - [\gamma - P_t \theta + P_t \pi \omega + P_t \Theta c_T]$$

$$= (1 - P_t \pi) \omega - (1 - P_t) \lambda^T$$

since the full payoff for capital becomes certain, where $\lambda^T$ is the marginal utility of income given the choice to kill in period $T$. This change is positive for sufficiently large reward for capital in the afterlife. Since consumption is constant in all periods, this may imply that savings were accumulated for spending on capital in period $\tau$.

For use in subsequent equations, we note

$$s_T^2 = \gamma \gamma - 2 \gamma \theta + 2 \gamma \omega - 2 \theta \omega + \theta \theta + \omega \omega + 2 [\gamma - \theta + \omega] \Theta c_T + \Theta \Theta c_T c_T$$

and

$$s_t^2 = \gamma \gamma - 2 P_t \gamma \theta + 2 P_t \gamma \pi \omega - 2 P_t \pi \theta \pi \omega + P_t P_t \omega \omega + P_t P_t \pi \pi \omega$$

$$+ 2 P_t [\gamma - P_t \theta + P_t \pi \omega] \Theta c_T + P_t P_t \Theta \Theta c_T c_T$$

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The lifetime budget constraint from the perspective of the first-period is

\[
\sum_{t=1}^{\tau} \Phi_t y_t \geq \sum_{t=1}^{\tau} \Phi_t [c_t + s_t - s_{t-1}|_{t>1}]
\]

\[
= \sum_{t=1}^{\tau-1} \Phi_t [c_t + s_t - s_{t-1}|_{t>1}] + \Phi_\tau [c_\tau + s_\tau - s_{\tau-1}|_{\tau>1}]
\]

\[
= \gamma + \sum_{t=1}^{\tau-1} \Phi_t [[1 + (P_t - P_{t-1}) \Theta] c_t - (P_t - P_{t-1}) \theta + (P_t - P_{t-1}) \pi \omega] + \Phi_\tau [[1 + (1 - P_{T-1}) \Theta] c_\tau - (1 - P_{T-1}) \theta + (1 - P_{T-1} \pi) \omega]
\]

After collecting terms, we have

\[
\sum_{t=1}^{\tau} \Phi_t y_t = \gamma + c_\tau \sum_{t=1}^{\tau} [\Phi_t [1 + (P_t - P_{t-1}) \Theta]]
\]

\[
+ \sum_{t=1}^{\tau-1} \Phi_t (P_t - P_{t-1}) [-\theta + \pi \omega] + \Phi_\tau [- (1 - P_{T-1}) \theta + (1 - P_{T-1} \pi) \omega]
\]

By solving for consumption we find

\[
c_\tau = \frac{\sum_{t=1}^{\tau} \Phi_t y_t - \gamma - [-\theta + \pi \omega] \sum_{t=1}^{\tau-1} \Phi_t (P_t - P_{t-1}) - [-\theta + \omega] \Phi_\tau (1 - P_{T-1})}{\sum_{t=1}^{\tau} \Phi_t [1 + (P_t - P_{t-1}) \Theta]}
\]

(10)

Lifetime utility for the would-be martyr, written in terms of optimal consumption, is

\[
\mathcal{L}_0(s_1, A_1) = \sum_{t=1}^{\tau-1} \Phi_t \left[ \theta c_t - \frac{\Theta}{2} c_t^2 + (\gamma + P_t \pi \omega) [s_t] - \frac{1}{2} [s_t]^2 + P_t \pi B - P_t (1 - \pi) D \right]
\]

\[
+ \Phi_\tau (1 + P_t \Theta) \Phi_\tau \left[ \frac{\Theta}{2} c_\tau^2 + \Phi_\tau \gamma [s_\tau] - \Phi_\tau \frac{1}{2} [s_\tau]^2 + \Phi_\tau \sigma + \Phi_\tau \Sigma + \Phi_\tau B + \Phi_\tau \omega [s_\tau] - \Phi_\tau \frac{\Theta}{2} c_\tau \Phi_\tau \right]
\]

\[
= \left[ \theta c_t - \frac{\Theta}{2} c_t^2 \right] \sum_{t=1}^{\tau-1} \Phi_t [1 + P_t \Theta] + \Phi_\tau [1 + \Theta]
\]

\[
+ \sum_{t=1}^{\tau-1} \Phi_t \left[ \frac{1}{2} \gamma \gamma - \frac{1}{2} P_t \pi \omega \theta + \frac{1}{2} P_t \pi \pi \omega \omega + P_t \gamma \pi \omega + P_t \pi B - (1 - \pi) P_t D \right]
\]

\[
+ \Phi_\tau \left[ \frac{1}{2} \gamma \gamma - \theta \theta + \frac{1}{2} \pi \omega \omega + \gamma \omega + B + \sigma + \Sigma \right]
\]
3.4 The Two-Period Case

We now turn to a two-period case of the $T$-period model, where we have in mind two stages of adulthood. We consider three relevant cases: no suicide, suicide by the young, and suicide by the old. Note that death is assured at the end of the second stage ($P_2 = 1$).

3.4.1 Case: No Killing

We begin with the case of no killing, where $K_1 = K_2 = 0$. By applying the consumption function for the $T$-period case, given as Equation 8, we find optimal consumption according to

\[
c^0 \equiv c_2 = \frac{y_1 + \Phi_2 y_2 - \gamma - P_1 \left( -\theta + \pi \omega \right) - \Phi_2 [1 - P_1] \left( -\theta + \pi \omega \right)}{[1 + P_1 \Theta] + \Phi_2 [1 + \Theta - P_1 \Theta]} \\
= \frac{y_1 + \Phi_2 y_2 - \gamma - \Phi_2 \left[ P_1 + P_1 \left( -\theta + \pi \omega \right) \right]}{[1 + P_1 \Theta] + \Phi_2 [1 + \Theta - P_1 \Theta]} 
\]

When there is no killing in either period, the optimal capital-accumulation path is

\[
s_1 = [1 - P_1] \gamma + P_1 s_2 
\]

In terms of optimal consumption, the optimal levels of religious capital are

\[
s_t = \gamma + P_t \left[ -\theta + \pi \omega + \Theta c^0 \right] \\
= \frac{[1 + \Phi_2 + \Phi_2 \Phi_2 \Theta] \gamma + P_t [1 + \Phi_2] \left( -\theta + \pi \omega \right) + P_t \Theta [y_1 + \Phi_2 y_2]}{1 + \Phi_2 + [\Phi_2 + P_t \Theta] \Theta} 
\]

We write lifetime utility in the 2-period case as a function of optimal consumption:

\[
\mathcal{L}_0 (s_1, A_1) = [\pi B - (1 - \pi) D] + \gamma \pi \omega \\
+ \frac{1}{2} [\gamma \gamma - P_t P_1 \theta \Theta + P_1 P_1 \pi \omega \pi \omega] + \frac{1}{2} \Phi_2 [\gamma \gamma - \theta \Theta + \pi \omega \pi \omega] \\
+ \left[ \theta c^0 - \frac{\Theta}{2} c^0 c^0 \right] [1 + P_t P_1 \Theta + \Phi_2 [1 + \Theta]]
\]

3.4.2 Case: Killing When Young

We next consider killing in the first period, so that $K_1 = 1$. In the case of suicidal terrorism committed in the first stage of life, we apply Equation 10 to find the consumption function:

\[
c^1 \equiv c_1 = \frac{y_1 - \gamma + \theta - \omega}{1 + \Theta} 
\]
In the chosen final period $T = 1$, the optimal balance between consumption and investment in religious capital is

$$s_1 = (\gamma - \theta + \omega) + \Theta c_1$$

$$= (\gamma - \theta + \omega) + \Theta \frac{y_1 - \gamma + \theta - \omega}{1 + \Theta}$$

It is easy to verify that optimal savings are zero so that the budget constraint holds. Lifetime utility in terms of optimal consumption is

$$\mathcal{L}_1(s_1, A_1) = \left[ \theta c_1^2 - \frac{\Theta}{2} c_1^2 \right] [1 + \Theta] + \frac{1}{2} \gamma \gamma - \frac{1}{2} \theta \theta + \frac{1}{2} \omega \omega + \gamma \omega + B + \sigma + \Sigma$$

### 3.4.3 Case: Killing When Old

Finally, we consider killing in the second period, so that $K_2 = 1$. In the case of suicidal terrorism committed in the second stage of life, we again apply Equation 10 to find optimal consumption:

$$c_2 = \frac{y_1 + \Phi_2 y_2 - \gamma + P_1 [\theta - \pi \omega] + \Phi_2 (1 - P_1) [\theta - \omega]}{[1 + P_1 \Theta] + \Phi_2 [1 + (1 - P_1) \Theta]}$$

When the agent decides to kill in the second period, the optimal capital-accumulation path is

$$s_1 = (1 - P_1) \gamma - P_1 (1 - \pi) \omega + P_1 s_2$$

In the chosen final period 2, the optimal balance between consumption and investment in religious capital is

$$s_2 = (\gamma - \theta + \omega) + \Theta c_2$$

$$= (\gamma - \theta + \omega) + \Theta c_2$$

The optimal capital stock in period 1 is determined as

$$s_1 = \gamma - P_1 \theta + P_1 \pi \omega + P_1 \Theta c_2$$

$$= \gamma - P_1 \theta + P_1 \pi \omega + P_1 \Theta c_2$$

Conditional on killing in the second period, lifetime utility as a function of consumption is:

$$\mathcal{L}_2(s_1, A_1) = \left[ \theta c_2^2 - \frac{\Theta}{2} c_2^2 \right] [1 + P_1 \Theta] + \Phi_2 \left[ 1 + \Theta \right]$$

$$+ \left[ \frac{1}{2} \gamma \gamma - \frac{1}{2} P_1 \theta \theta + \frac{1}{2} P_1 P_1 \pi \pi \omega \omega + P_1 \gamma \pi \omega + P_1 \pi B - P_1 (1 - \pi) D \right]$$

$$+ \Phi_2 \left[ \frac{1}{2} \gamma \gamma - \frac{1}{2} \theta \theta + \frac{1}{2} \omega \omega + \gamma \omega + B + \sigma + \Sigma \right]$$
3.5 Consumption

In this and following sections, we consider mainly the two-period model. We found optimal policies for consumption, savings, and religious-capital investment, conditional on the decision of whether and when to kill. Before proceeding, we derive derivatives and limits for the three consumption functions presented above. These will be useful in the next section, where we will consider the agent’s choice of whether and when to terrorize.

We derived the consumption function in the case of no killing

\[ c^0 = \frac{y_1 + \Phi_2 y_2 - \gamma + [P_1 + \Phi_2 (1 - P_1)] (\theta - \pi \omega)}{[1 + P_1 \Theta] + \Phi_2 [1 + (1 - P_1) \Theta]} \]

In the case of killing while young, we found the consumption function

\[ c^1 = \frac{y_1 - \gamma + (\theta - \omega)}{1 + \Theta} \]

Finally, we found the consumption function conditional on the decision to kill in the second stage of life:

\[ c^2 = \frac{y_1 + \Phi_2 y_2 - \gamma + P_1 [\theta - \pi \omega] + \Phi_2 (1 - P_1) [\theta - \omega]}{[1 + P_1 \Theta] + \Phi_2 [1 + (1 - P_1) \Theta]} \]

We see that \( c^0 \) is very similar to \( c^2 \); they differ only in the expected probability of gaining entry to heaven if death occurs in the second period. When entry is certain, the expected benefit of religious capital in the afterlife increases, and so consumption will decrease to allow greater investment spending. We have the result

\[ c^0 \geq c^2 \]

with equality if \( \pi = 1 \) or \( \omega = 0 \). With decreasing marginal utility of consumption, this implies \( \partial E_2 / \partial c^2 \geq \partial E_0 / \partial c^0 \), all else equal.

Recall the first-order conditions \( \theta - \Theta c^i = \lambda^i \) where \( \lambda^i \) is the marginal utility of income for the cases \( i \in \{0, 1, 2\} \).

The derivatives of consumption with respect to expected income are

\[ MPC^0 = \frac{\partial c^0}{\partial y_1} = \frac{\partial c^0}{\partial P_2 y_2} = \frac{1}{[1 + P_1 \Theta] + \Phi_2 [1 + (1 - P_1) \Theta]} \]

\[ MPC^1 = \frac{\partial c^1}{\partial y_1} = \frac{1}{1 + \Theta} \]

\[ MPC^2 = \frac{\partial c^2}{\partial y_1} = \frac{\partial c^2}{\partial P_2 y_2} = \frac{1}{[1 + P_1 \Theta] + \Phi_2 [1 + (1 - P_1) \Theta]} \]

Consumption spending clearly increases with income, where the marginal propensity to consume is a function of the probability of death and the curvature parameter for consumption. In the case of \( c^1 \), death is certain and so natural
mortality rates are not relevant. Note that \( \frac{\partial c}{\partial y_2} = \Phi_2 MPC \) increases with the survival rate, so that potential income in the later stage holds little value if the agent does not expect to live. Note too that \( MPC^0 = MPC^2 \), though the consumption functions are not identical.

Consumption spending falls when religious capital provides greater reward in the natural life:

\[
\frac{\partial c^i}{\partial \gamma} = -MPC^i
\]

where \( i \in \{0, 1, 2\} \).

Consumption also falls as religious capital is valued more in the afterlife. Except when entry is certain because of martyrdom, the effect increases as confidence in gaining entry (\( \pi \)) increases.

\[
\frac{\partial c^0}{\partial \omega} = -MPC^0 \times [P_1 + \Phi_2 (1 - P_1)] \pi
\]
\[
\frac{\partial c^1}{\partial \omega} = -MPC^1
\]
\[
\frac{\partial c^2}{\partial \omega} = -MPC^2 \times [P_1 \pi + \Phi_2 (1 - P_1)]
\]

We see that changes in confidence have a corresponding effect on consumption spending such that consumption falls as the probability of gaining entry to heaven increases. Since entry into heaven is believed certain as a reward for suicide, the impact of the changes in the entry probability are zero when martyrdom is chosen before natural death can occur.

\[
\frac{\partial c^0}{\partial \pi} = -MPC^0 \times [P_1 + \Phi_2 (1 - P_1)] \omega
\]
\[
\frac{\partial c^1}{\partial \pi} = 0
\]
\[
\frac{\partial c^2}{\partial \pi} = -MPC^2 \times P_1 \omega
\]

Finally, we find the limit for consumption spending as the mortality rate in the first period goes to zero

\[
c^0\big|_{P_1 \to 0} = \frac{y_1 + y_2 - \gamma + \theta - \pi \omega}{2 + \Theta}
\]
\[
c^1\big|_{P_1 \to 0} = \frac{y_1 - \gamma + \theta - \omega}{1 + \Theta}
\]
\[
c^2\big|_{P_1 \to 0} = \frac{y_1 + y_2 - \gamma + \theta - \omega}{2 + \Theta}
\]

and as the mortality becomes certain

\[
c^0\big|_{P_1 \to 1} = c^2\big|_{P_1 \to 1} = \frac{y_1 - \gamma + \theta - \pi \omega}{1 + \Theta}
\]
\[
c^1\big|_{P_1 \to 1} = \frac{y_1 - \gamma + \theta - \omega}{1 + \Theta}
\]
Given the decision to end life in the first period, the probability of death from other causes has no effect on consumption. As the probability of natural death in the first period vanishes, we see that the consumption function \( c^2 \) becomes similar to that of the no-killing case, though consumption is somewhat lower since entry is assured. As first-period mortality becomes certain, consumption behavior becomes identical such that \( c^0 = c^2 \) but \( c^1 \) remains unchanged. As the probability of natural death approaches unity, we see that the consumption function is very similar to the case of killing in the first period, but consumption is higher since entry to heaven is not assured.

In earlier sections, we found equations that specify optimal investment in religious capital in terms of consumption, and these functions together with the budget constrain specify optimal savings. Thus, these results for consumption readily may be employed to find corresponding results for other economic behavior.

### 3.6 Regularity Conditions

We noted earlier that the quadratic model requires regularity conditions in order for it to represent standard economic theory. Here we list several of those restrictions, and we impose the following set of conditions upon the model and parameters. For later use, we will consider each set of restrictions for the case of zero first-period mortality and certain first-period mortality.

#### 3.6.1 Nonnegative consumption

Since consumption must be nonnegative in each period of life, the consumption functions provide us with restrictions on the parameters. The restrictions for the cases of no killing, first-period martyrdom, and second-period martyrdom, are

\[
\begin{align*}
0 & \leq y_1 + \Phi_2 y_2 - \gamma + [P_1 + \Phi_2 \Phi_2] (\theta - \pi \omega) \\
0 & \leq y_1 - \gamma + \theta - \omega \\
0 & \leq y_1 + \Phi_2 y_2 - \gamma + [P_1 + \Phi_2 \Phi_2] \theta - [P_1 \pi + \Phi_2 \Phi_2] \omega
\end{align*}
\]

When first-period mortality rates vanish \((P_1 \to 0)\), we find

\[
\begin{align*}
0 & \leq y_1 + y_2 - \gamma + \theta - \pi \omega \\
0 & \leq y_1 - \gamma + \theta - \omega \\
0 & \leq y_1 + y_2 - \gamma + \theta - \omega
\end{align*}
\]

We see that if the second restriction holds then so does the third, and if the third holds then so does the first. Therefore, only the second may bind. When
first-period mortality is certain, \((P_1 \to 1)\), then
\[
\begin{align*}
0 & \leq y_1 - \gamma + \theta - \pi \omega \\
0 & \leq y_1 - \gamma + \theta - \omega \\
0 & \leq y_1 - \gamma + \theta - \pi \omega
\end{align*}
\]
We see that if the second restriction holds then so does the first and third. Again, only the second may bind. In summary, the relevant nonnegativity constraint for consumption, regardless of the extreme value for mortality, is
\[
y_1 - \gamma + \theta - \omega \geq 0
\]

### 3.6.2 Non-decreasing marginal utility

Since our quadratic utility function is not monotonic, in contrast to standard microeconomic consumer models, we must ensure that the chosen level of consumption is sufficiently small so that utility is non-decreasing. From the first-order conditions, we found marginal utility of consumption as \(\lambda^i = \theta - \Theta c^i\), for cases \(i \in \{0, 1, 2\}\). The corresponding nonnegativity conditions \((\lambda = \theta - \Theta c \geq 0)\) may be written
\[
\begin{align*}
\frac{\theta}{\Theta} & \geq c^0 = \frac{y_1 + \Phi_2 y_2 - \gamma + [P_1 + \Phi_2 \Phi_2] (\theta - \pi \omega)}{[1 + P_1 \Theta] + \Phi_2 [1 + \Phi_2 \Theta]} \\
\frac{\theta}{\Theta} & \geq c^1 = \frac{y_1 - \gamma + \theta - \omega}{1 + \Theta} \\
\frac{\theta}{\Theta} & \geq c^2 = \frac{y_1 + \Phi_2 y_2 - \gamma + P_1 [\theta - \pi \omega] + \Phi_2 [\theta - \omega]}{[1 + P_1 \Theta] + \Phi_2 [1 + \Phi_2 \Theta]}
\end{align*}
\]
In the extreme case that early mortality is certain \((P_1 \to 0)\), we have
\[
\begin{align*}
0 & \geq y_1 + y_2 - \gamma - \pi \omega - \frac{2\theta}{\Theta} \\
0 & \geq y_1 - \gamma - \omega - \frac{\theta}{\Theta} \\
0 & \geq y_1 + y_2 - \gamma - \omega - \frac{2\theta}{\Theta}
\end{align*}
\]
Note that if the first condition holds, then so does the third. When mortality is certain \((P_1 \to 1)\), then
\[
\begin{align*}
0 & \geq y_1 - \gamma - \pi \omega - \frac{\theta}{\Theta} \\
0 & \geq y_1 - \gamma - \omega - \frac{\theta}{\Theta} \\
0 & \geq y_1 - \gamma - \pi \omega - \frac{\theta}{\Theta}
\end{align*}
\]
Note that if the first and third conditions hold, then so does the second. In summary, the relevant regularity conditions for nonnegative marginal utility are, for \( P_1 \to 0 \),

\[
0 \geq y_1 + y_2 - \gamma - \pi \omega - \frac{2\theta}{\Theta} \\
0 \geq y_1 - \gamma - \omega - \frac{\theta}{\Theta}
\]

and for \( P_1 \to 1 \) are

\[
0 \geq y_1 - \gamma - \pi \omega - \frac{\theta}{\Theta}
\]

### 3.6.3 Nonnegative investment

We also assume that stocks of religious capital must be non-negative (\( s \geq 0 \)). The non-negativity constraints for the first-period are

\[
s_1^0 = \gamma - P_1\theta + P_1 \pi \omega + P_1 \Theta c^0 \\
       = \gamma - P_1\theta + P_1 \pi \omega + P_1 \Theta \frac{y_1 + \Phi_2 y_2 - \gamma + [P_1 + \Phi_2 \Phi_2] (\theta - \pi \omega)}{1 + P_1 \Theta} \geq 0
\]

\[
s_1^1 = \gamma - \theta + \omega + \Theta c_1 \\
       = \gamma - \theta + \omega + \Theta \frac{y_1 - \gamma + \theta - \omega}{1 + \Theta} \geq 0
\]

\[
s_1^2 = \gamma - P_1\theta + P_1 \pi \omega + P_1 \Theta c^2 \\
       = \gamma - P_1\theta + P_1 \pi \omega + P_1 \Theta \frac{y_1 + \Phi_2 y_2 - \gamma + P_1 [\theta - \pi \omega] + \Phi_2 (1 - P_1) [\theta - \omega]}{1 + P_1 \Theta} \geq 0
\]

When \( P_1 \to 0 \), we have

\[
\gamma \geq 0 \\
\gamma - \theta + \omega + \Theta y_1 \geq 0 \\
\gamma \geq 0
\]

When first-period mortality rates are zero, then the first and second constraints are relevant. When \( P_1 \to 1 \),

\[
\gamma - \theta + \pi \omega + \Theta y_1 \geq 0 \\
\gamma - \theta + \omega + \Theta y_1 \geq 0 \\
\gamma - \theta + \pi \omega + \Theta y_1 \geq 0
\]

When mortality is certain, we see that if the first constraint holds, then so do the others. In summary, we have the following relevant constraints to ensure nonnegative investment spending: when \( P_1 \to 0 \),

\[
\gamma \geq 0 \\
\gamma - \theta + \omega + \Theta y_1 \geq 0
\]

and when \( P_1 \to 1 \),

\[
\gamma - \theta + \pi \omega + \Theta y_1 \geq 0
\]
3.6.4 Irreversible investment

Nonnegativity of investment spending and zero depreciation rates imply irreversibility and non-declining capital stocks \((s_2 \geq s_1)\). The second-period constraints may be written as

\[
\begin{align*}
   s_2^0 &= \gamma - \theta + \pi \omega + \Theta c^0 \\
   &= \gamma - \theta + \pi \omega + \Theta \frac{y_1 + \Phi_2 y_2 - \gamma + [P_1 + \Phi_2 \Phi_2] (\theta - \pi \omega)}{[1 + P_1 \Theta] + \Phi_2 [1 + (1 - P_1) \Theta]} \\
   s_2^1 &= 0 \\
   s_2^2 &= \gamma - \theta + \omega + \Theta c^2 \\
   &= \gamma - \theta + \omega + \Theta \frac{y_1 + \Phi_2 y_2 - \gamma + P_1 [\theta - \pi \omega] + \Phi_2 \Phi_2 [\theta - \omega]}{[1 + P_1 \Theta] + \Phi_2 [1 + \Phi_2 \Theta]} \\
   &\geq s_1^0
\end{align*}
\]

After introducing the optimal first-period investment equations and simplifying, we find

\[
\begin{align*}
   (1 - P_1) \left[ -\theta + \pi \omega + \Theta \frac{y_1 + \Phi_2 y_2 - \gamma + [P_1 + \Phi_2 \Phi_2] (\theta - \pi \omega)}{[1 + P_1 \Theta] + \Phi_2 [1 + (1 - P_1) \Theta]} \right] &\geq 0 \\
   (1 - P_1) \left[ -\theta + \omega + \Theta \frac{y_1 + \Phi_2 y_2 - \gamma + P_1 [\theta - \pi \omega] + \Phi_2 \Phi_2 [\theta - \omega]}{[1 + P_1 \Theta] + \Phi_2 [1 + \Phi_2 \Theta]} \right] &\geq 0
\end{align*}
\]

For \(P_1 \to 0\), we have

\[
\begin{align*}
   -\theta (2 + \Theta) + 2\pi \omega + \Theta |y_1 + y_2 - \gamma + \theta| &\geq 0 \\
   s_2^1 &= 0 \\
   -\theta (2 + \Theta) + 2\omega + \Theta |y_1 + y_2 - \gamma + \theta| &\geq 0
\end{align*}
\]

We see that if the first constraint holds, then so does the third. The second constraint holds by definition. When \(P_1 \to 1\), we have

\[
\begin{align*}
   s_2^0 &= 0 \\
   s_2^1 &= 0 \\
   s_2^2 &= 0
\end{align*}
\]

In summary, the relevant constraint for irreversible investment is

\[
-\theta (2 + \Theta) + 2\pi \omega + \Theta |y_1 + y_2 - \gamma + \theta| \geq 0
\]

In conclusion, the relevant nonnegativity constraint for consumption, regardless of the extreme value assumed for mortality, is

\[
y_1 - \gamma + \theta - \omega \geq 0
\]

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The relevant regularity conditions for nonnegative marginal utility are, when \( P_1 \to 0 \)
\[
0 \geq y_1 + y_2 - \gamma - \pi \omega - \frac{2\theta}{\Theta} \\
0 \geq y_1 - \gamma - \omega - \frac{\theta}{\Theta}
\]
and when \( P_1 \to 1 \)
\[
0 \geq y_1 - \gamma - \pi \omega - \frac{\theta}{\Theta}
\]
We have the following relevant constraints to ensure nonnegative investment spending when \( P_1 \to 0 \)
\[
\gamma \geq 0 \\
\gamma - \theta + \omega + \Theta y_1 \geq 0
\]
and when \( P_1 \to 1 \)
\[
\gamma - \theta + \pi \omega + \Theta y_1 \geq 0
\]
The relevant constraint for irreversible investment is
\[
-\theta (2 + \Theta) + 2\pi \omega + \Theta |y_1 + y_2 - \gamma + \theta| \geq 0
\]

3.7 The Decision to Kill

We found optimal policies for consumption, savings, and religious-capital investment, conditional on the decision of whether and when to kill. To determine if or when the individual will terrorize, we must rank utility levels for the cases. We now compare the cases and characterize conditions for when the agent will decide to terrorize. We then consider how changes in beliefs and circumstances may affect these choices.

3.7.1 Modeling The Decision

This decision is a form of the optimal stopping problem. The agent chooses whether to plan an intentional ending of life at a particular point or whether to leave death to chance. Of course, premature death may disrupt planned killing later in life. If the agent decides to kill and carries out the plans successfully, then the agent and his community reap the rewards and consequences of the decision.

Depict discounted lifetime utility in the no-killing case as \( L^0 \), in the \( Y = 1 \) case as \( L^1 \), and in the \( Y = 2 \) case as \( L^2 \). We characterize the advantage of choice \( i \) over choice \( j \) as \( \Delta_{ij} \equiv L^i - L^j \). For the two-period problem, we then calculate three relevant differences: \( \Delta_{10} \equiv L^1 - L^0 \), \( \Delta_{20} \equiv L^2 - L^0 \), and \( \Delta_{21} \equiv L^2 - L^1 \). The agent will choose not to kill if \( \Delta_{10} < 0 \) and \( \Delta_{20} < 0 \), to kill in the first period if \( \Delta_{10} > 0 \) and \( \Delta_{21} < 0 \), and to kill in the second period if \( \Delta_{20} > 0 \) and \( \Delta_{21} > 0 \).
After employing the lifetime utility equations presented earlier and then simplifying and collecting terms, we find the suicide decision as a quadratic function of consumption and the parameters, where in turn consumption is a function of income and other parameters. The perceived advantage to self and community of killing while young over not killing is

\[
\Delta_{10} = \left[ \theta c^1 - \frac{\Theta}{2} c^1 c^1 \right] [1 + \Theta] - \left[ \theta c^0 - \frac{\Theta}{2} c^0 c^0 \right] \left[ [1 + P_1 P_1 \Theta] + \Phi_2 [1 + \Theta] \right] - \frac{1}{2} \Phi_2 [\gamma \gamma + P_1 \theta \theta] + \frac{1}{2} [1 - (P_1 P_1 + \Phi_2) \pi \pi] \omega \omega \\
+ (1 - \pi) [\gamma \omega + B + D] + \sigma + \Sigma
\]

while the advantage of killing when old versus not killing is

\[
\Delta_{20} = \left[ \theta c^2 - \frac{\Theta}{2} c^2 c^2 \right] - \left[ \theta c^0 - \frac{\Theta}{2} c^0 c^0 \right] \left[ [1 + P_1 P_1 \Theta] + \Phi_2 [1 + \Theta] \right] + \Phi_2 \left[ (1 - \pi) \left[ \frac{1 + \pi}{2} \omega \omega + \gamma \omega + B - D \right] + \sigma + \Sigma \right]
\]

and the advantage of killing when old over killing while young is

\[
\Delta_{21} = \left[ \theta c^2 - \frac{\Theta}{2} c^2 c^2 \right] \left[ [1 + P_1 P_1 \Theta] + \Phi_2 [1 + \Theta] \right] - \left[ \theta c^1 - \frac{\Theta}{2} c^1 c^1 \right] [1 + \Theta] + \frac{1}{2} \Phi_2 [\gamma \gamma + P_1 \theta \theta] - \frac{1}{2} P_1 (1 - P_1 \pi \pi) \omega \omega \\
- P_1 (1 - \pi) [\gamma \omega + B + D] + \sigma + \Sigma
\]

Clearly, suicide presents greater advantage to the individual with higher anticipated benefit to the natural self and the community (\(\sigma\)) and with greater expected reward in the afterlife (\(\Sigma\)). Early suicide provides a means to ensure the basic benefits of heaven, where damnation follows natural death with perceived probability \((1 - \pi)\) and where early natural death occurs with probability \(P_1\). As the probability of early death increases and as the expected likelihood of damnation increases, martyrdom while young grows more attractive, ignoring any secondary effects of these changes through consumption. We derive these and more results in the following section.

### 3.7.2 Marginal Effects and Extreme Conditions

While the decision about whether and when to commit suicide is a discrete choice and presents a discontinuity for the model, the difference between two conditional lifetime utility equations is continuous, where each lifetime utility sum is conditional on the suicide choice. Hence, we can calculate derivatives of the functions \(\Delta\) with respect to the parameters of the model to determine the predicted effects on the decision to terrorize.

We also consider the effects of extreme views and conditions. To keep the algebra simple, we will consider two primary cases: short life spans, with certain
death after the first period, and long lives. For both, we will consider the implications of other extremes. We begin by deriving the optimal decision rules under varying mortality rates. Mortality rates may be determined by health, economic well-being, and safety. As the first-period mortality rate approaches zero, we see that the perceived advantage of terrorizing in the first period is

$$\Delta_{10|P_1=0} = -\theta [y_2 + \omega (1 - \pi)] - \frac{\Theta}{2} \left[ \frac{y_1 - \gamma + \theta - \omega}{1 + \Theta} - \frac{y_1 + y_2 - \gamma + \theta - \pi \omega}{2 + \Theta} \right]$$

$$-\frac{1}{2} \gamma \gamma + (1 - \pi) \left[ \frac{1 + \pi}{2} \omega \omega + \gamma \omega + B + D \right] + \sigma + \Sigma$$

and for terrorizing in the second period is

$$\Delta_{20|P_1=0} = - (1 - \pi) \left[ \frac{\omega}{2 + \Theta} [2 \theta (1 + \Theta) + \Theta [y_1 + y_2] + (1 + \Theta) (1 + \pi) \omega + 2 \gamma] + B + D \right] + \sigma + \Sigma$$

and for terrorizing later in life is

$$\Delta_{21|P_1=0} = \theta y_2 - \frac{\Theta}{2} \left[ \frac{[y_1 + y_2 - \gamma + \theta - \omega]^2}{2 + \Theta} - \frac{[y_1 - \gamma + \theta - \omega]^2}{1 + \Theta} \right] + \frac{1}{2} \gamma \gamma$$

If we ignore differences utility caused by differences in the levels of consumption, then terrorism when early death is unlikely is driven mainly by the desire to avoid damnation and to gain the earthly and heavenly rewards for terrorism. Terrorism will be delayed if the earthly benefits of religious capital are large.

As mortality becomes certain for the young, the agent may have little to lose and potentially much to gain through terrorism. We see that the advantage to the individual of martyrdom is to ensure entry to heaven, plus natural benefits of killing, plus afterlife benefits.

$$\Delta_{10|P_1=1} = \frac{1 - \pi}{1 + \Theta} \left[ -\theta + \Theta y_1 + \gamma + \frac{1 + \pi}{2} \omega \right] \omega + (1 - \pi) [B + D] + \sigma + \Sigma$$

$$\Delta_{20|P_1=1} = 0$$

$$\Delta_{21|P_1=1} = \frac{1 - \pi}{1 + \Theta} \left[ -\theta + \Theta y_1 + \gamma + \frac{1 + \pi}{2} \omega \right] \omega - (1 - \pi) [B + D] - \sigma - \Sigma$$

**Income** We now consider the implications of varying income levels. We consider two extremes. First, we consider zero expected lifetime income. We next consider income sufficiently high to drive the marginal utility of income to zero; this is the highest level of income that is consistent with the model.

When mortality known to be low and income is zero, then we expect that consumption will be zero so that the budget constraints hold. We assume that consumption and religious investment remain nonnegative. We find that optimal
consumption becomes \( c^0 = \frac{-\gamma + \theta - \pi \omega}{2 + \Theta} \), \( c^1 = \frac{-\gamma + \theta - \omega}{1 + \Theta} \), and \( c^2 = \frac{-\gamma + \theta - \omega}{2 + \Theta} \). For our nonnegativity assumptions to hold both for consumption and for investment, then \( \gamma = 0, \theta = \pi \omega, \) and \( \theta = \omega. \) For these conditions to hold simultaneously with \( \pi > 0, \) then \( \gamma \to 0, \theta \to 0, \) and \( \omega \to 0. \) These strong and perhaps undesirable implications for the parameters are a result of our choice of quadratic utility. In general, there is no theoretical reason that the parameters must vanish along with income. However, that consumption and investment should disappear when income vanishes is reasonable. The resulting martyrdom equations when mortality rates are zero become

\[
\Delta_{10}|_{p_1=0, y=0} = (1 - \pi) [B + D] + \sigma + \Sigma
\]
\[
\Delta_{20}|_{p_1=0, y=0} = (1 - \pi) [B + D] + \sigma + \Sigma
\]
\[
\Delta_{21}|_{p_1=0, y=0} = 0
\]

Of course, if consumption truly were zero, then mortality quickly would follow. Instead, we assume assume a subsistence level of consumption that is sufficient to sustain life but that is insufficient to allow accumulation of religious capital. In this case, martyrdom would be driven by the desire to ensure salvation and to gain reward for martyrdom. There is no consideration of religious capital, since there are no funds available to invest. If salvation is certain, then all depends on the expected net benefit of killing.

When mortality is expected to be low and income is high, then we expect that consumption and investment will be large. The greatest level of income that remains consistent with the regularity conditions is \( y_1 = \gamma + \omega + \frac{\theta}{\Theta} \) and \( y_1 + y_2 = \gamma + \pi \omega + \frac{2 \theta}{\Theta}. \) After combining these conditions, we see that the constraints on income are \( y_1 = \gamma + \omega + \frac{\theta}{\Theta} \) and \( y_2 = \frac{\theta}{\Theta} - \omega (1 - \pi). \) Given these income levels and zero first-period mortality rates, we find that optimal consumption becomes \( c^0 = c^1 = \frac{\theta}{\Theta} \) and \( c^2 = \frac{\theta}{\Theta} - \frac{(1 - \pi) \omega}{2 + \Theta}. \) The perceived advantages of killing are

\[
\Delta_{10}|_{p_1=0, y=\infty} = -\frac{1}{2} \frac{\theta \Theta}{\Theta} - \frac{1}{2} \gamma \gamma + (1 - \pi) \left[ \frac{1 + \pi + \pi \Theta}{2 + \Theta} \omega \omega + \gamma \omega + B + D \right] + \sigma + \Sigma
\]
\[
\Delta_{20}|_{p_1=0, y=\infty} = (1 - \pi) \left[ \frac{1 + \pi + \pi \Theta}{2 + \Theta} \omega \omega + \gamma \omega + B + D \right] + \sigma + \Sigma
\]
\[
\Delta_{21}|_{p_1=0, y=\infty} = \frac{\theta \Theta}{2 \Theta} + \frac{1}{2} \gamma \gamma - \Theta \frac{(1 - \pi) (1 - \pi)}{2 (2 + \Theta)} \omega \omega
\]

With excess funds available to invest, natural and supernatural returns for religious capital again becomes relevant. Greater natural reward for capital and more funds to invest makes longer life more desirable, while the greater investment makes the anticipated supernatural reward more attractive and thus may make suicide more attractive.

When early death is certain and income is low, then consumption is given by \( c^0 = c^1 = c^2 = \frac{y_1 - \gamma + \omega}{1 + \Theta} \) and the decision rules are
\[
\begin{align*}
\Delta_{10}|_{t=1, y=0} &= (1 - \pi) [B + D] + \sigma + \Sigma \\
\Delta_{20}|_{t=1, y=0} &= 0 \\
\Delta_{21}|_{t=1, y=0} &= -(1 - \pi) [B + D] - \sigma - \Sigma
\end{align*}
\]

Once again, martyrdom decisions are driven entirely by the desire to escape damnation and to gain natural and supernatural rewards for martyrdom. When income is high, then \( c^0 = c^2 = \frac{\theta}{\sigma} + \frac{(1 - \pi) \omega}{1 + \Theta} \) and \( c^1 = \frac{\theta}{\sigma} \) and the decision rules are determined by

\[
\begin{align*}
\Delta_{10}|_{t=1, y=\infty} &= (1 - \pi) \left[ \frac{2\Theta + 1 + \pi}{2(1 + \Theta)} \omega \omega + \gamma \omega + B + D \right] + \sigma + \Sigma \\
\Delta_{20}|_{t=1, y=\infty} &= 0 \\
\Delta_{21}|_{t=1, y=\infty} &= -(1 - \pi) \left[ \frac{2\Theta + 1 + \pi}{2(1 + \Theta)} \omega \omega + \gamma \omega + B + D \right] - \sigma - \Sigma
\end{align*}
\]

With much to invest and natural demise pending, much depends on anticipation of investment returns in the afterlife, on the relative bliss of heaven, and on expected payoff for a martyr’s death.

We also calculate the marginal effects of changes to income. According to the model, the suicide decisions vary with first-period income as

\[
\begin{align*}
\frac{\partial \Delta_{10}}{\partial y_1} &= \left[ \theta - \Theta c^1 \right] [1 + \Theta] \frac{\partial c^1}{\partial y_1} - \left[ \theta - \Theta c^0 \right] \left[ 1 + P_1 P_1 \Theta \right] + \Phi_2 \left[ 1 + \Theta \right] \frac{\partial c^0}{\partial y_1} \\
&= -\Theta \left[ c^1 - c^0 \right]
\end{align*}
\]

and

\[
\begin{align*}
\frac{\partial \Delta_{20}}{\partial y_1} &= \left[ \theta - \Theta c^2 \right] \frac{\partial c^2}{\partial y_1} - \left[ \theta - \Theta c^0 \right] \left[ 1 + P_1 P_1 \Theta \right] + \Phi_2 \left[ 1 + \Theta \right] \\
&= -\Theta \left[ c^2 - c^0 \right]
\end{align*}
\]

and

\[
\begin{align*}
\frac{\partial \Delta_{21}}{\partial y_1} &= \left[ \theta - \Theta c^2 \right] \left[ 1 + P_1 P_1 \Theta \right] + \Phi_2 \left[ 1 + \Theta \right] - \left[ \theta - \Theta c^1 \right] \frac{\partial c^1}{\partial y_1} \left[ 1 + \Theta \right] \\
&= -\Theta \left[ c^2 - c^1 \right]
\end{align*}
\]

In each case, the model implies that terrorism becomes less attractive as it requires greater sacrifice of consumption. We examined the consumption functions in the section above; it is difficult to say, in general, whether the levels of these equations are positive or negative and whether the inclination to terrorize theoretically increases or decreases with first-period income.

Krueger [13] argues that income is not related in an obvious way with the decision to commit suicidal terrorism. In order to reconcile this finding with the
model, the derivative of $\Delta$ with respect to first-period income must be small. When will this be true? It will be true when the differences between optimal consumption levels are small. This might be true under various conditions. One condition leading to similar consumption functions is a high first-period mortality rate. After filling in optimal consumption levels, we find

$$\frac{\partial \Delta_{10}}{\partial y_1} \Big|_{P_1=1} = \frac{\Theta}{1 + \Theta} (1 - \pi) \omega$$

$$\frac{\partial \Delta_{20}}{\partial y_1} \Big|_{P_1=1} = 0$$

$$\frac{\partial \Delta_{21}}{\partial y_1} \Big|_{P_1=1} = -\frac{\Theta}{1 + \Theta} (1 - \pi) \omega$$

We see that if religious capital is not valued much in the afterlife or if the entrance probability is high, then optimal consumption levels are similar and first-period income does not affect substantially the terrorism decision when the mortality rate is high. Mortality rates may be high either because of poor health or because of risky activities chosen by the agent, such as armed conflict. Recall that we assumed that religious capital has no effect on the entry probability $\pi$. While it remains to be verified, we might speculate that doubt of one’s faith would weaken the perceived probability of damnation and lessen the gain found in martyrdom by a factor of $(1 - p^N)$, where $p^N$ is the perceived probability of nonexistence and $(1 - p^N)(1 - \pi)$ is the perceived probability of damnation. If this reasoning is correct, then doubt would weaken the appeal of martyrdom and make more likely the finding of small first-period income effects. Recall too that we required expected solvency, so that martyrs cannot leave debt; this is a strong assumption that unduly may affect the model results.

We find that expected second-period income affects the martyrdom decision as

$$\frac{\partial \Delta_{10}}{\partial \Phi_{2y2}} = - \left[ \theta - \Theta c^0 \right] \left[ \left[ 1 + P_1 P_1 \Theta \right] \Phi_2 \left[ 1 + \Theta \right] \right] \frac{\partial c^0}{\partial \Phi_{2y2}}$$

$$= - \left[ \theta - \Theta c^0 \right]$$

$$= -\lambda^0$$

and

$$\frac{\partial \Delta_{20}}{\partial \Phi_{2y2}} = \left[ \theta - \Theta c^2 \right] \frac{\partial c^2}{\partial \Phi_{2y2}} - \left[ \theta - \Theta c^0 \right] \frac{\partial c^0}{\partial \Phi_{2y2}} \left[ \left[ 1 + P_1 \Phi_2 \Theta \right] + \Phi_2 \left[ 1 + \Theta \right] \right]$$

$$= -\Theta \left[ c^2 - c^0 \right]$$

$$= \frac{\partial \Delta_{20}}{\partial y_1}$$
and
\[
\frac{\partial \Delta_{21}}{\partial \Phi_2 y_2} = \left[ \theta - \Theta c^2 \right] \left[ \left[ 1 + P_1 P_1 \Theta + \Phi_2 \left[ 1 + \Theta \right] \right] \frac{\partial c^2}{\partial \Phi_2 y_2} - \left[ \theta - \Theta c^1 \right] \left[ 1 + \Theta \right] \frac{\partial c^1}{\partial \Phi_2 y_2} \right] = \theta - \Theta c^2 = \lambda^2
\]

where \( \lambda^i \) is the marginal utility of income in the optimization problem for case \( i \). Given the assumption that consumption increases monotonically, \( \lambda \) must be nonnegative, and so the decision to kill while young becomes less attractive as expected second-period income and utility increase. In the consideration of whether to kill when old, the potential gain in utility is affected by income according to the difference in consumption levels. Recall that we found \( c^0 \geq c^2 \) with equality if \( \pi = 0 \) or \( \omega = 0 \). This implies \( \frac{\partial \Delta_{20}}{\partial y_1} = \frac{\partial \Delta_{20}}{\delta \Phi_2 y_2} \geq 0 \).

Recall that optimal investment in religious capital is related to optimal consumption, and we simply have chosen to write the \( \Delta \) functions in terms of consumption rather than in terms of the parameters. By replacing the consumption terms with income and other parameters, we would find the derivatives as functions of income and other parameters. Again, scholars report little evidence that income is related to terrorism, and so we might expect that this derivative should be small. The equations indicate that income will have little effect on terrorist activity if the marginal utility of expected second-period income is small. This would be true if the agent anticipates the approach of a point of satiation, where greater consumption provides little additional benefit. Note the related finding that \( \frac{\partial \Delta_{20}}{\partial y_2} = -\Phi_2 \lambda^0 \), so that a low survival rate will leave the terrorist unresponsive to potential income later in life.

**Entry to Heaven** We next consider the implications of beliefs about the likelihood of gaining entry to heaven. First, we consider the case of certain damnation following a life certain to be long, where entry to heaven is possible only through martyrdom. The personal benefit of killing is given by

\[
\begin{align*}
\Delta_{10} \big|_{P_1=0, \pi=0} &= -\theta \left[ y_2 + \omega \right] - \frac{\Theta}{2} \left[ \frac{\left[ y_1 - \gamma + \theta - \omega \right]^2}{1 + \Theta} - \frac{\left[ y_1 + y_2 - \gamma + \theta \right]^2}{2 + \Theta} \right] \\
&\quad - \frac{1}{2} \gamma \gamma + \frac{1}{2} \omega \omega + \gamma \omega + B + D + \sigma + \Sigma \\
\Delta_{20} \big|_{P_1=0, \pi=0} &= -\omega \left[ \theta + \frac{\Theta}{2 + \Theta} \left[ \left[ y_1 + y_2 - \gamma + \theta + \frac{1}{2} \omega \right] \right] + \frac{1}{2} \omega \omega + \gamma \omega + B - D + \sigma + \Sigma \\
\Delta_{21} \big|_{P_1=0, \pi=0} &= \theta y_2 - \frac{\Theta}{2} \left[ \frac{\left[ y_1 + y_2 - \gamma + \theta - \omega \right]^2}{2 + \Theta} - \frac{\left[ y_1 - \gamma + \theta - \omega \right]^2}{1 + \Theta} \right] + \frac{1}{2} \gamma \gamma
\end{align*}
\]
Next, we consider the case when life is sure to be long and entry to heaven is certain.

\[
\Delta_{10}|_{\pi_0, \pi_1} = -\theta y_2 - \frac{\Theta}{2} \left[ \frac{(y_1 - \gamma + \theta - \omega)^2}{1 + \Theta} - \frac{(y_1 + y_2 - \gamma + \theta - \omega)^2}{2 + \Theta} \right] - \frac{1}{2} \gamma \gamma + \sigma + \Sigma
\]

\[
\Delta_{20}|_{\pi_0, \pi_1} = \sigma + \Sigma
\]

\[
\Delta_{21}|_{\pi_0, \pi_1} = \theta y_2 - \frac{\Theta}{2} \left[ \left( \frac{(y_1 + y_2 - \gamma + \theta - \omega)^2}{2 + \Theta} \right) - \left( \frac{(y_1 - \gamma + \theta - \omega)^2}{1 + \Theta} \right) \right] + \frac{1}{2} \gamma \gamma
\]

When early death is assured and damnation is sure to follow, the decision rules are determined according to

\[
\Delta_{10}|_{\pi_1, \pi_0} = \frac{1}{1 + \Theta} \left[ -\theta + \Theta y_1 + \gamma + \frac{1}{2} \omega \right] \omega + B + D + \sigma + \Sigma
\]

\[
\Delta_{20}|_{\pi_1, \pi_0} = 0
\]

\[
\Delta_{21}|_{\pi_1, \pi_0} = -\frac{1}{1 + \Theta} \left[ -\theta + \Theta y_1 + \gamma + \frac{1}{2} \omega \right] \omega - B - D - \sigma - \Sigma
\]

Finally, we consider the case when early mortality and entry to heaven are certain.

\[
\Delta_{10}|_{\pi_1, \pi_1} = \sigma + \Sigma
\]

\[
\Delta_{20}|_{\pi_1, \pi_1} = 0
\]

\[
\Delta_{21}|_{\pi_1, \pi_1} = -\sigma - \Sigma
\]

We find the following marginal impacts of higher anticipated entry probabilities.

\[
\frac{\partial \Delta_{10}}{\partial \pi} = [P_1 P_1 + \Phi_2] \lambda^0 \omega - [P_1 P_1 + \Phi_2] \pi \omega \omega - \gamma \omega - B - D
\]

\[
\frac{\partial \Delta_{20}}{\partial \pi} = -[\lambda^2 P_1 - \lambda^0 [P_1 + \Phi_2 \Phi_2]] \omega - \Phi_2 [\pi \omega \omega + \gamma \omega + B + D]
\]

\[
\frac{\partial \Delta_{21}}{\partial \pi} = P_1 [\lambda^2 \omega + P_1 \pi \omega \omega + \gamma \omega + B + D]
\]

In the earlier examination of optimal consumption, we found that higher entrance likelihood led to lower consumption as the agent invests more in religious capital. For this reason, higher entry probabilities tend to make early martyrdom look more attractive than no killing when the marginal utility of consumption is high in the no-killing case. Greater entry probabilities make martyrdom less attractive as potential afterlife rewards for religious capital increase and as the benefit of heaven relative to hell increases. Finally, higher entry probabilities reduce potential loss for delayed martyrdom when first-period mortality rates are high. In general, the model implies that the tendency to terrorize varies with the probability of gaining entry to heaven without resorting to martyrdom, but theory does not indicate the level or slope of this effect.
Afterlife Reward for Religious Capital  We next consider the implications of beliefs about reward in the afterlife for religious capital. We consider two extremes. First, we consider the belief that there is no reward for religious capital in the afterlife. Next, we consider the expectation that rewards are sufficient to drive the agent to asceticism, which we approximate as zero consumption.

As mortality approaches zero and expected reward in the afterlife for religious capital is zero, then we expect that consumption will increase. We assume that marginal utility and religious investment remain nonnegative. We find that optimal consumption becomes:

\[ c^0 = c^2 = \frac{y_1 + y_2 - \gamma + \theta}{2 + \Theta} \]

and for terrorizing in the second period is

\[ c^0 = c^2 = \frac{y_2}{2 + \Theta} \]

and for terrorizing later in life is

\[ c^0 = (1 - \pi) \frac{y_1 - \gamma + \theta}{2 + \Theta} \]

When mortality known to be low and expected reward in the afterlife for religious capital is very high, then we expect that consumption will be sacrificed for investment. The greatest reward that remains consistent with the regularity conditions is \( y_1 - \gamma + \theta = \omega \). We find that optimal consumption becomes:

\[ c^0 = \frac{(1 - \pi) y_1 - \gamma + \theta}{2 + \Theta} \]

\[ c^1 = 0 \]

\[ c^2 = \frac{y_2}{2 + \Theta} \]

The perceived advantage of first-period martyrdom is

\[ \Delta_{10}|_{P_1 \to 0, \omega - 0} = -\theta y_2 - \frac{\Theta}{2} \left[ \frac{[y_1 - \gamma + \theta]^2}{1 + \Theta} - \frac{[y_1 + y_2 - \gamma + \theta]^2}{2 + \Theta} \right] - \frac{1}{2} \gamma \gamma + (1 - \pi) [B + D] + \sigma + \Sigma \]

and for terrorizing in the second period is

\[ \Delta_{20}|_{P_1 \to 0, \omega - 0} = -(1 - \pi) [B + D] + \sigma + \Sigma \]

and for terrorizing later in life is

\[ \Delta_{21}|_{P_1 \to 0, \omega - 0} = \theta y_2 - \frac{\Theta}{2} \left[ \frac{[y_1 + y_2 - \gamma + \theta]^2}{2 + \Theta} - \frac{[y_1 - \gamma + \theta]^2}{1 + \Theta} \right] + \frac{1}{2} \gamma \gamma \]
and for terrorizing later in life is
\[
\Delta_{21}\big|_{P_1 \to 0, \omega \to \infty} = \theta y_2 - \frac{\Theta}{2} \frac{y_2 y_2}{2 + \Theta} + \frac{1}{2} \gamma \gamma
\]

When an early death is certain and religious capital has no value in the afterlife, optimal consumption is given as \(c^0 = c^1 = c^2 = \frac{21 - \gamma}{1 + \Theta}\). The decision rules for martyrdom are determined by
\[
\begin{align*}
\Delta_{10}\big|_{P_1 \to 1, \omega \to 0} &= (1 - \pi) [B + D] + \sigma + \Sigma \\
\Delta_{20}\big|_{P_1 \to 1, \omega \to 0} &= 0 \\
\Delta_{21}\big|_{P_1 \to 1, \omega \to 0} &= - (1 - \pi) [B + D] - \sigma - \Sigma
\end{align*}
\]

When religious capital brings great reward in the afterlife, such that the regularity condition for nonnegative consumption just holds and \(\omega = y_1 - \gamma + \theta\), then
\[
\begin{align*}
\Delta_{10}\big|_{P_1 \to 1, \omega \to \infty} &= \frac{1 - \pi}{1 + \Theta} \left[ \frac{2 \Theta + 1 + \pi}{2} y_1 - \frac{1 - \pi}{2} (\theta - \gamma) \right] [y_1 - \gamma + \theta] + (1 - \pi) [B + D] + \sigma + \Sigma \\
\Delta_{20}\big|_{P_1 \to 1, \omega \to \infty} &= 0 \\
\Delta_{21}\big|_{P_1 \to 1, \omega \to \infty} &= \frac{1 - \pi}{1 + \Theta} \left[ \frac{2 \Theta + 1 + \pi}{2} y_1 - \frac{1 - \pi}{2} (\theta - \gamma) \right] [y_1 - \gamma + \theta] - (1 - \pi) [B + D] - \sigma - \Sigma
\end{align*}
\]

We find influences on terrorism with respect to change in anticipated heavenly reward for religious capital. The advantage of killing when young increases as
\[
\frac{\partial \Delta_{10}}{\partial \omega} = -\lambda^1 + \lambda^0 \left[ P_1 P_1 + \Phi_2 \right] \pi + \left[ 1 - \left[ P_1 P_1 + \Phi_2 \right] \pi \pi \right] \omega + (1 - \pi) \gamma
\]

and the advantage of killing when old increases with \(\omega\) as
\[
\begin{align*}
\frac{\partial \Delta_{20}}{\partial \omega} &= -\lambda^2 \left[ P_1 \pi + \Phi_2 \Phi_2 \right] + \lambda^0 \left[ P_1 + \Phi_2 \Phi_2 \right] \pi \\
&+ \Phi_2 (1 - \pi) (1 + \pi) \omega + \Phi_2 (1 - \pi) \gamma
\end{align*}
\]

and, finally, the advantage of waiting to kill increases as
\[
\frac{\partial \Delta_{21}}{\partial \omega} = -\lambda^2 \left[ P_1 \pi + \Phi_2 \Phi_2 \right] - \lambda^1 - P_1 (1 - P_1 \pi \pi) \omega - P_1 (1 - \pi) \gamma
\]
For given levels of consumption and marginal utility, greater reward in the afterlife makes terrorism more attractive as natural-world reward for religious capital increases. Once again, theory does not indicate the sign or slope of this derivative. The slopes for terrorism versus peace more likely are positive if religious capital is valued highly in the natural life, but the result for the third equation more likely becomes negative as it becomes more favorable to lock in gain.

**Natural Reward for Religious Capital**  Terrorism appears more attractive to the young when religious capital is valued highly in the afterlife but less favorable when capital is valued in the natural life. The tendency will fall with survival rates and with the perceived entry probability. If consumption levels are similar, then the effect of natural reward for religious capital depends on whether the additional period of utility stemming from religious capital outweighs the possible loss of reward if entry to heaven is denied.

\[ \frac{\partial \Delta_{10}}{\partial \gamma} = \Theta \left[ c^1 - c^0 \right] - \Phi_2 \gamma + (1 - \pi) \omega \]

In the second case, when death is certain and there may be little to lose, then for moderate differences in the levels of consumption, the effect of higher reward surely is positive if the expected afterlife reward is large and the entry probability is positive.

\[ \frac{\partial \Delta_{20}}{\partial \gamma} = \Theta \left[ c^2 - c^0 \right] + \Phi_2 (1 - \pi) \omega \]

Since \( c^0 \geq c^2 \) and \( \omega \geq 0 \), the effects of higher preferences is diminished somewhat because of the indirect loss of utility through lower consumption levels. Still, once we employ the consumption functions, we find that the sign of the derivative unambiguously is nonnegative.

\[ \frac{\partial \Delta_{20}}{\partial \gamma} = \Phi_2 (1 - \pi) \left[ \frac{1 + \Phi_2 + P_1 \Theta}{1 + P_1 \Theta} + \frac{\Phi_2}{1 + (1 - P_1) \Theta} \right] \omega \]

Finally, terrorism late in life appears more favorable when religious capital is valued highly in the natural life and when the probability of dying young and being condemned is low. For higher mortality rates and greater likelihood of damnation, then increased preference for religious capital leads the agent toward terrorism at a younger age lest he lose the afterlife reward and spend eternity in hell.

\[ \frac{\partial \Delta_{21}}{\partial \gamma} = \Theta \left[ c^2 - c^1 \right] + \Phi_2 \gamma - P_1 (1 - \pi) \omega \]

**Afterlife Reward for Martyrdom**  We next consider the implications of afterlife rewards for martyrdom. Here we consider only the reward presented after admission to heaven; we do not consider the effect of martyrdom on the
entrance probability. When life certainly will be long and there is no afterlife reward for killing, the decision rules are determined according to

\[
\Delta_{10}|_{P_1=0, \Sigma=0} = \left[ \theta c^1 - \frac{\Theta}{2} c^1 c^1 \right] [1 + \Theta] - \left[ \theta c^0 - \frac{\Theta}{2} c^0 c^0 \right] [(1 + P_1 P_1 \Theta) + \Phi_2 [1 + \Theta]]
\]

\[
\quad - \frac{1}{2} \Phi_2 [\gamma \gamma + P_1 \theta \theta] + \frac{1}{2} [1 - (P_1 P_1 + \Phi_2) \pi \pi] \omega \omega
\]

\[
\quad + (1 - \pi) [\gamma \omega + B + D] + \sigma
\]

and

\[
\Delta_{20}|_{P_1=0, \Sigma=0} = \left[ \theta c^2 - \frac{\Theta}{2} c^2 c^2 \right] [2 + \Theta]
\]

\[
\quad + \left[ (1 - \pi) \left( \frac{1 + \pi}{2} \omega \omega + \gamma \omega + B + D \right) + \sigma \right]
\]

and

\[
\Delta_{21}|_{P_1=0, \Sigma=0} = \left[ \theta c^2 - \frac{\Theta}{2} c^2 c^2 \right] [(1 + P_1 P_1 \Theta) + \Phi_2 [1 + \Theta]] - \left[ \theta c^1 - \frac{\Theta}{2} c^1 c^1 \right] [1 + \Theta] + \frac{1}{2} [\gamma \gamma + P_1 \theta \theta]
\]

When life is expected to be long and rewards go to infinity, then so too does the expected advantage of killing.

\[
\Delta_{10}|_{P_1=0, \Sigma=\infty} \rightarrow \infty
\]

\[
\Delta_{20}|_{P_1=0, \Sigma=\infty} \rightarrow \infty
\]

\[
\Delta_{21}|_{P_1=0, \Sigma=\infty} = \theta y_2 - \frac{\Theta}{2} \left[ \frac{[y_1 + y_2 - \gamma + \theta - \omega]^2}{2 + \Theta} - \frac{[y_1 - \gamma + \theta - \omega]^2}{1 + \Theta} \right] + \frac{1}{2} [\gamma \gamma + P_1 \theta \theta]
\]

When the agent expects to die young and anticipates no reward in the afterlife for martyrdom, then his decision rules are given by the following.

\[
\Delta_{10}|_{P_1=1, \Sigma=0} = \frac{\Theta}{2} \left[ \frac{[y_1 - \gamma + \theta - \omega]^2}{1 + \Theta} - \frac{[y_1 - \gamma + \theta - \pi \omega]^2}{1 + \Theta} \right]
\]

\[
\quad + (1 - \pi) \left[ \frac{1 + \pi}{2} \omega \omega + \gamma \omega - \omega \theta + B + D \right] + \sigma
\]

\[
\Delta_{20}|_{P_1=1, \Sigma=0} = 0
\]

\[
\Delta_{21}|_{P_1=1, \Sigma=0} = \frac{\Theta}{2} \left[ \frac{[y_1 - \gamma + \theta - \pi \omega]^2}{1 + \Theta} - \frac{[y_1 - \gamma + \theta - \omega]^2}{1 + \Theta} \right]
\]

\[
\quad - (1 - \pi) \left[ \frac{1 + \pi}{2} \omega \omega + \gamma \omega - \omega \theta + B + D \right] + \sigma
\]

Finally, when life is short and martyrs are rewarded greatly in the afterlife, then
The perceived advantage of killing increases directly with the anticipated net expected value of the act. The marginal benefit is

\[
\frac{\partial \Delta_{10}}{\partial \Sigma} = 1 \\
\frac{\partial \Delta_{20}}{\partial \Sigma} = \Phi_2 \\
\frac{\partial \Delta_{21}}{\partial \Sigma} = -P_1
\]

In summary, we see that without anticipated reward in the afterlife for killing, suicidal terrorism is likely to be viewed optimal only for high natural rewards or if damnation seems too probable. When reward in the afterlife is expected to be very large, then killing is seen as optimal, and killing sooner removes the possibility of missing the opportunity and perhaps facing condemnation.

**Natural Reward for Martyrdom** The decision rules are affected by reward in the natural world in similar fashion. Recall that we assumed that the agent has no doubt about his faith. With doubt, the anticipated afterlife rewards would be discounted accordingly but the anticipated natural rewards would not be discounted. If we also allowed the possibility of failure in the execution of the suicidal mission, then the expected natural rewards likewise might be adjusted by the probability of success.

With high mortality and the expectation that suicidal terrorism will bring retribution to community, severe pain to self, or other great loss of social welfare, then the decision rules follow these equations:

\[
\Delta_{10}|_{p_1=0, \Sigma \rightarrow -\infty} \rightarrow -\infty \\
\Delta_{20}|_{p_1=0, \Sigma \rightarrow -\infty} \rightarrow -\infty \\
\Delta_{21}|_{p_1=0, \Sigma \rightarrow -\infty} = \left[\theta c^2 - \frac{\Theta}{2} c^2 e^2\right] \left[(1 + P_1 P_1 \Theta) + \Phi_2 (1 + \Theta)\right] - \left[\theta c^1 - \frac{\Theta}{2} c^1 e^1\right] \left[1 + \Theta\right] + \frac{1}{2} \left[\gamma \gamma + P_1 \theta \theta\right]
\]

When martyrs expect to gain fame, fortune for their families, revenge over their enemies, or other great natural reward, then the perceived advantages of terrorism are these.

\[
\Delta_{10}|_{p_1=0, \Sigma \rightarrow -\infty} \rightarrow \infty \\
\Delta_{20}|_{p_1=0, \Sigma \rightarrow -\infty} \rightarrow \infty \\
\Delta_{21}|_{p_1=0, \Sigma \rightarrow -\infty} = \theta y_2 - \frac{\Theta}{2} \left[\frac{[y_1 + y_2 - \gamma + \Theta - \omega]^2}{2 + \Theta} - \frac{[y_1 - \gamma + \Theta - \omega]^2}{1 + \Theta}\right] + \frac{1}{2} \left[\gamma \gamma + P_1 \theta \theta\right]
\]
When life is expected to be short and natural costs for suicide are high, the decision rules follow

\[
\Delta_{10}|_{p_1 \to -1, \sigma \to -\infty} \to -\infty \\
\Delta_{20}|_{p_1 \to -1, \sigma \to -\infty} = 0 \\
\Delta_{21}|_{p_1 \to -1, \sigma \to -\infty} \to \infty
\]

and when natural rewards are high, the rules are determined by

\[
\Delta_{10}|_{p_1 \to -1, \sigma \to \infty} \to \infty \\
\Delta_{21}|_{p_1 \to -1, \sigma \to \infty} = 0 \\
\Delta_{21}|_{p_1 \to -1, \sigma \to \infty} \to -\infty
\]

The perceived advantage of killing increases directly with the anticipated net expected value of the act. The marginal effects are

\[
\frac{\partial \Delta_{10}}{\partial \sigma} = 1 \\
\frac{\partial \Delta_{20}}{\partial \sigma} = \Phi_2 \\
\frac{\partial \Delta_{21}}{\partial \sigma} = -p_1
\]

In summary, sufficiently great punishment in the natural world for malevolent activity makes suicidal terrorism seem undesirable. Similarly, sufficiently great earthly reward makes a martyr’s death attractive, and greater likelihood of natural death will move up the optimal timing of suicide.

**Heaven and Hell** The perceived advantage of killing increases directly with the perceived pleasure of heaven and agony of hell, though discounted by the perceived likelihood of damnation.

\[
\frac{\partial \Delta_{10}}{\partial B} = \frac{\partial \Delta_{10}}{\partial D} = (1 - \pi) \\
\frac{\partial \Delta_{20}}{\partial B} = \frac{\partial \Delta_{20}}{\partial D} = \Phi_2 (1 - \pi) \\
\frac{\partial \Delta_{21}}{\partial B} = \frac{\partial \Delta_{21}}{\partial D} = -p_1 (1 - \pi)
\]

This implies that the decisions of an agent certain of his salvation will not be affected by the basic utility of heaven or hell. If salvation is not certain, then higher mortality rates will make the agent inclined to kill sooner as the benefit of escaping damnation increases.

### 3.8 Calibration

In this section, we examine the model from a different angle. We now specify values for various parameters and evaluate the equations. Note that we currently
employ no real data or estimates, but instead we simply choose parameter values to illustrate the model. This analysis currently is very limited, though future extensions may allow us to confront the model with actual data and to extend the model using numerical techniques.

3.8.1 The Utility Function

To illustrate the utility functions and the martyrdom decision, we choose a set of parameters for the model. We allow the expected afterlife reward for martyrdom ($\Sigma$) to vary. The parameters are chosen to meet the regularity conditions and so that each possible martyrdom decision will be optimal for a certain range of reward values. The parameters are not assumed to be realistic and the results should not be interpreted as a meaningful representation of the world. In the following graph, we see that no killing is optimal when rewards are small. For somewhat greater reward, killing becomes optimal when natural death is inevitable anyway. For large anticipated reward, immediate martyrdom is seen as desirable, and so even the young will choose to kill and to die.

![Lifetime Utility Graph](image)

3.8.2 The Marginal Propensity to Consume

We now take a closer look at consumption behavior as specified in the model. Recalling the equation for the $MPC$, we solve for $\Theta$:

$$MPC^0 = \frac{1}{1 + P_1 \Theta} + \frac{\Phi_2}{1 + (1 - P_1) \Theta}$$

$$\Theta = \frac{1}{\Phi_2 + P_1 P_1} \left[ \frac{1}{MPC^0} - 1 - \Phi_2 \right]$$

Suppose that the $MPC^0 = 0.5$ and that $P_1 \in \{0.1, 0.9\}$. Then according to the model and conditional on our calibrated values, $\Theta \in \{0.10989011, 0.989010989\}$.

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The same results will hold for $MPC^2$. Suppose we apply the assumed mortality rates and the calculated values for $\Theta$ to the model of first-period suicide. We find that $MPC^1 \in \{0.900990099, 0.502762431\}$. This suggests that someone choosing to die when natural death is unlikely would tend to consume much of any increase in available resources, with the rest going to investment in religious capital and none devoted to savings. If natural death is likely anyway, then economic behavior for the terrorist will not be unusual, at least in terms of $MPC$, since the value is changed little from the other cases.

In Figure 3.8.2 we display the marginal propensity to consume out of expected income while allowing the probability of death to vary, given that $\Theta = 0.1$. We see that the MPC increases with the probability of death in the cases of no killing or second-period suicide. In the case of first-period suicide, the MPC is constant.

In Figure 3.8.2 we display the marginal propensity to consume out of expected income while allowing the probability of death to vary, given that $\Theta = 1.0$. We again see that the MPC increases with the probability of death in the cases of no killing or second-period suicide, while the MPC is constant in the case of first-period suicide. In all cases, MPC is lower than in the case with $\Theta = 0.1$. In Figure 3.8.2 we again display the marginal propensity to consume while allowing the probability of death to vary, given that $\Theta = 10.0$. We see that $MPC$ initially increases with the probability of death in the cases of no killing or second-period suicide, but then $MPC$ falls for high probability of death. In the case of first-period suicide, the $MPC$ is constant. In all cases, $MPC$ is lower than in the cases with lower $\Theta$.

Remaining work will summarize descriptions of terrorists provided in the literature and use those findings to restrict the model parameters, and we will compare the model predictions to the evidence. Several possibilities are these:
Terrorism is not obviously dependent on income. Krueger [13, Table 2.4] reports no association between GDP per capita and terrorism, though these findings are not specific to suicidal terrorism. The model thus implies that differences in consumption should be small (approximately $c^0 = c^1 = c^2$).

Terrorists often relatively wealthy and well educated (approximately $y_1 + \Phi_2 y_2 \to \infty$). Krueger [13, Table 2.4] reports no association between literacy and terrorism.

Terrorists seldom cite afterlife concerns for activities. This might imply that they expect no significant afterlife reward or change in entry probabilities ($\Sigma \to 0$, $\tau \to \infty$). It also might imply that the expected natural reward simply might overshadow other considerations. This could be true, for example, if revenge were a strong motivator. See Berman [2].

Terrorists typically cite revenge and other temporal concerns as primary motives ($\sigma \to \infty$). Krueger [13, Table 2.4] reports strong associations between GDP per capita of the target country and terrorism, between countries that occupy and become targets of terror, and between occupied countries and sources of terrorism.

3.9 Analysis and Policy Implications

We now review and summarize the implications of the model. We relate these findings to the empirical evidence reported in the literature. Finally, we will consider feasible means by which conditions might be altered by human influence such that terrorism is not preferred.
3.9.1 Results

The model implies that wealthy individuals who are confident of salvation are less likely to terrorize. The individual who otherwise has little hope of entry to heaven and little income to consume or invest in religious capital is more likely to kill because he has less to lose and relatively more to gain.

**Income** The marginal effects of changes to income are given by relative consumption levels. The model implies that terrorism becomes less attractive as it requires greater sacrifice of consumption. It is difficult to say, in general, whether these effects are positive or negative and whether the inclination to terrorize theoretically increases or decreases with income.

When income is low, then martyrdom is driven by the desire to ensure salvation and to gain natural and supernatural rewards for martyrdom. High income allows high consumption and investment, which in turn makes longer life more desirable, though the greater capital stock makes the anticipated supernatural reward more attractive and thus may make suicide more attractive. Thus, when income is high, much depends on anticipation of investment returns in the afterlife, on the relative bliss of heaven, and on expected payoff for a martyr’s death.

We find that income later in life will have little effect on terroristic activity if the marginal utility of expected second-period income is small, either due to satiation to low survival rates.

**Entry to Heaven** Higher entrance likelihood leads to lower consumption as the agent invests more in religious capital, and so marginal utility of consumption increases. Greater entry probabilities make martyrdom less attractive as
potential afterlife rewards for religious capital increase and as the benefit of heaven relative to hell increases. Finally, higher entry probabilities reduce potential loss for delayed martyrdom when first-period mortality rates are high. The model implies that the tendency to terrorize varies with the probability of gaining entry to heaven without resorting to martyrdom, but it does not provide a clear indication of how changes in the likelihood of entry affects the decision to terrorize.

**Afterlife Reward for Religious Capital**  For given levels of consumption, greater reward in the afterlife makes terrorism more attractive as natural-world reward for religious capital increases. Once again, theory does not indicate the sign of this derivative, though increasing violence is more likely if religious capital also is valued highly in the natural life. Only if higher investment drives the marginal utility of consumption very high will the derivatives with respect to afterlife returns likely become negative.

**Natural Reward for Religious Capital**  For given levels of consumption, the effect of greater natural return on religious investment increases with expected returns in the afterlife. This appeal of afterlife reward makes terrorism more appealing, but higher natural returns may lead the terrorist to delay execution of his plan.

**Afterlife Reward for Martyrdom**  We find that without anticipated reward in the afterlife for killing, suicidal terrorism is likely to be viewed optimal only for high natural rewards or if damnation seems too probable. When reward in the afterlife is expected to be very large, then killing is seen as optimal, and killing sooner removes the possibility of missing the opportunity and perhaps facing condemnation.

**Natural Reward for Martyrdom**  Sufficiently great punishment in the natural world for malevolent activity, whether due to retribution to the community, physical pain of death, or otherwise, makes suicidal terrorism seem undesirable. Similarly, sufficiently great earthly reward makes a martyr’s death attractive, such as fame or financial reward to family, and greater likelihood of natural death will move up the optimal timing of suicide.

**Heaven and Hell**  The perceived advantage of killing increases directly with the perceived pleasure of heaven and agony of hell, though discounted by the perceived likelihood of damnation. The decisions of an agent certain of his salvation will not be affected by the basic utility of heaven or hell. If salvation is not certain, then higher mortality rates will make the agent inclined to kill sooner as the benefit of escaping damnation increases.

Finally, these results imply clearly that religious beliefs should affect decisions, including the decision of whether to kill. Berman and others report little evidence that anticipation of the afterlife drives decisions to kill. An explanation
for why expectations about the afterlife might not seem to matter is hyperbolic
discounting, where utility in the afterlife is discounted more heavily than utility
in the natural life. Also, any doubt about faith also would serve to discount
expected utility in the next existence more heavily. In addition, one might as-
sume that the proportional increase in utility for activity in the natural world is
small; in other words, life in heaven will be so grand that extra treasure would
provide little additional utility. If the agent is certain of salvation, then this too
would weaken any inclination to accept lower utility in this world to gain utility
in the next.

3.9.2 Policy

We consider potential means by which policy makers might affect conditions
for potential terrorists such that they choose not to terrorize. Such policies
should take religious convictions seriously, recognizing the limited ability of
governments to influence perceptions of divine directives and anticipation of
reward or punishment in the afterlife. We anticipate the finding that investment
in religious capital can be substituted for terrorism, to a limited but possibly
useful degree, and that such substitution can be made optimal in the eyes of
the individual with well-crafted policies.

We now consider what policies might prove feasible to make the no-terrorism
case the preferred option. Of course, these policies merely are implied by the
model and might have little value in the real world. Before such recommenda-
tions are taken seriously, the model must be confronted thoroughly with empir-
ical data. The following simply suggest the sort of feasible actions that such a
model might imply effective.

Punish Terrorism We saw that sufficiently great punishment in the natural
world \((\sigma \to -\infty)\) will make the agent avoid terror. Whether feasible and ethical
means exist to create sufficiently severe punishment remains in question. The
challenge would be greater for an agent who derives great pleasure in the pain
of his enemy and for those who anticipate great supernatural reward.

Increase Future Income Another means to alter decisions is to subsidize
income. In particular, higher future income \((y_2 \to \infty)\) makes immediate suicide
less attractive. However, the model suggest that this will not prevent attacks by
the elderly who have little time left anyway. We might expect that this policy
could be used to delay terror, and that during the delay the agent might acquire
social and religious capital. This capital might be vulnerable following suicide
where otherwise it might be enjoyed by the surviving community, and so there
is greater opportunity to punish malevolent behavior.

Improve health The model is somewhat ambiguous about the effect of im-
proved health and safety \((P_1 \to 0)\).
Subsidize investment in Religious and Social Capital The model results were not clear in the effect of higher natural returns for investment in religious capital \((\gamma \to \infty)\), though again higher natural returns acts to postpone the optimal timing of terror. Perhaps there is no way to subsidize natural returns in a way that would satisfy the potential martyr. If such a means could be identified and implemented, then it might be used to discourage terror by the young, and perhaps it could be used in combination with other policies to discourage terrorism at any age.

Harden Potential Targets Finally, though the current model does not incorporate the possibility of a failed attack, we anticipate that the possibility of failure would reduce the appeal of an attack. This would be true for a reduced expected natural payoff (lower expected \(\sigma\)) and for reduced afterlife benefits following a failed attempt at martyrdom (lower entry probability or lower \(\Sigma\)).

4 Groups of Terrorists

Berman [2], Berman and Laitin ([4] and [3]), and others emphasize the importance of groups for the successful execution of terrorist attacks. They note that terrorists seldom act alone. Indeed, it is the exceptions that are memorable, such as the attacks of Dr. Theodore John "Ted" Kaczynski, who also is known as the Unabomber. Though actual attacks may be carried out by a single agent, that agent almost always was trained and supported, and possibly recruited, by a group.

Here we consider the interaction of multiple agents of the sort considered in the text above. We again consider the case of the religious agent who considers whether to engage in terrorism. We follow the lead of Berman and others in our notion of a religious terrorist group, though we focus on explicitly religious motivations.

Under what conditions will this agent remain loyal to his group and to his faith, and when will greed and other desires persuade him to defect? Does anything distinguish the terrorist with religious motives from the terrorist with secular concerns?

4.1 Whether to Defect

We consider first the case in which a young religious agent is a member of a group of religious terrorists. The group engages in a mission that carries religious implications. The agent is presented the choice either to remain loyal to the religious organization or to defect. Loyalty means that the agent will carry out the mission, earn income \(y_t\), and use a portion of the income to increase his religious capital stock by \(i_t\). Defection yields additional income \(y'_t\) but requires the loss of all religious capital so that \(s_t = 0\). Such assumed loss of religious capital may be reasonable if the mission is considered the work of God, and
where disobedience carries both loss of standing in the religious community and loss of reward in the afterlife.

Suppose that the terrorist group has \( N \) agents, and suppose that booty \( Y \) for the attack are distributed equally among members. Then we write the individual income level for the as \( y = \frac{Y}{N} \), if all members remain loyal, and income for a single defector is \( y' = Y = Ny \), where \( y' = \frac{N-1}{N}Y \). We assume that the entity under attack would be willing to offer a reward for defection up to the full value of the target, so that both the reward and the target value is \( Y \).

We further assume that if the disloyal agent survives the current period that he will have opportunity to seek redemption in future periods. Such redemption may come by investing economic wealth in religious capital. We do not distinguish normal income from ill-gotten gain, and so the profit for defection may be invested in future periods. We maintain the assumption of exogenous income and that defection does not affect the exogenous income stream, apart from the additional gains from defection, so that the second-period income level is not dependent on first-period decisions. We assume that savings or debt accumulated in the past will remain after defection. These assumptions are made for sake of simplicity and may be relaxed.

Berman [2] considers a similar case, drawing material from Berman and Laitin ([3] and [4]), but there the cost of defection presumably are material and social. We instead focus on the loss of religious capital. We reproduce the graph from Berman (Figure A.1) here:

Following the arguments in Berman, agents will remain loyal for low-value missions. The payoff for remaining loyal is high relative to the payoff for defecting, since there is no loss of social or religious capital. Under our model of religious agents, we suppose that the payoff for loyalty may be a combination of financial wealth and religious capital, where the capital may yield benefits in the natural world and beyond. The slope is fairly flat since any material gain and glory from the mission is distributed among group members. Alternatively, the agent might choose to defect, thus sacrificing his existing stock of social and religious capital but gaining a greater amount of financial wealth. The loss of capital presents a fixed cost for defection, and so the returns for low-value missions are smaller than for loyalty. Returns increase rapidly with the project value, since the defecting agent keeps the reward for defection rather than sharing the payoff with other group members.

Berman offers several examples of such cases. These include guarding a trade route in a lawless territory. After the group conquers a trade route, it posts its members along the route to collect tolls and to protect traders. Guards along the route are faced with the decision either to remain loyal and receive a share of the tolls or to defect, steal the trader’s truck and cargo, and flee, but also to face loss of home and community. A second example is a violent assault to gain geographic territory. Perhaps the territory includes an important trade route, and the group want to control trade and charge tolls to fund other operations. The group is divided in a coordinated attack on the present occupiers of the territory. Each subgroup must remain loyal for the attack to be successful.
The occupiers may offer each subgroup a bribe to cease fighting, thus causing the entire attack to fail. The decision for attacking group members is whether to remain loyal and reap a share of the reward or to accept a bribe and to defect, though defection may come at a high price. Finally, a conspirator in a suicide bombing plot faces the choice of whether to defect and report the plot, thereby collecting a reward for the information but sacrificing his standing in the community and possibly the execution of himself and his family. The other option is to remain loyal and share in the glory of a successful attack on the enemy. Rewards may be especially large in the last case, since suicide attacks typically are reserved for high-value targets.

In each of these cases, group leaders face a defection constraint. This constraint must be considered in the design of attacks and other operations. Berman emphasizes the importance of loyalty for the success of a terrorist group, to the extent that great loyalty is necessary in order for groups to be effective. Successful groups find ways to ensure that defection constraints are high, so that members will be reluctant to defect.

Again, our focus is on religious beliefs and values. We consider the implications of strong beliefs that may dominate other motivating factors. We do not argue that explicitly secular motivations are not important, but instead we
seek to understand the implications of religious beliefs and objectives. We will derive the optimal behavior and utility for the case of no defection. This closely follows the earlier work for the single-agent case. We consider only financial profit, which later may be used to rebuild religious capital, but we do not consider here any direct affect on capital for loyalty, such as the gain of respect among the religious community following a mission. We next derive optimal behavior for the case of defection, where the agent will reoptimize following the unexpected arrival of an opportunity to defect. Finally, we compare the two cases to determine when the agent optimally will choose to defect.

**No Defection** We begin with the case of loyalty. The decision rules for consumption, investment, and saving will be identical to those derived in the no-killing case for the single-agent model. In the planning stage at time period 0, the agent determines the optimal level of consumption in the first and second period to be

\[
c_1 = c_2 = \frac{y_1 + \Phi_2 y_2 - \gamma - [\Phi_2 + P_1 P_1] (-\theta + \pi \omega)}{[1 + P_1 \Theta] + \Phi_2 [1 + \Theta - P_1 \Theta]}
\]

If we assume that the regularity conditions hold so that the irreversibility of capital condition is not violated, then we can write the second-period budget constraint as

\[
A_1 + y_2 + s_1 \geq c_2 + s_2
\]

where \(A_1\) is asset holdings at the end of the first period. Recalling Equation 4, the optimal balance between second-period consumption and capital stock is

\[
s_2 = \Theta c_2 + [\gamma - \theta + \pi \omega]
\]

This is consistent with the savings rule established before the first period, since the agent has no reason to revise the balance established in the initial decision rules. With the budget constraint binding and employing the optimal balance between savings and consumption, we find second-period consumption to be

\[
c_2 = \frac{A_1 + y_2 + s_1}{1 + \Theta} - \frac{\gamma - \theta + \pi \omega}{1 + \Theta}
\]

Optimal second-period investment spending then is

\[
s_2 = \Theta \frac{A_1 + y_2 + s_1}{1 + \Theta} + \frac{\gamma - \theta + \pi \omega}{1 + \Theta}
\]

We can write the second-order terms as

\[
[c_2]^2 = \frac{[A_1 + y_2 + s_1]^2}{[1 + \Theta]^2} - \frac{2 [A_1 + y_2 + s_1] [\gamma - \theta + \pi \omega]}{[1 + \Theta]^2} + \frac{[\gamma - \theta + \pi \omega]^2}{[1 + \Theta]^2}
\]

\[
[s_2]^2 = \Theta \frac{[A_1 + y_2 + s_1]^2}{[1 + \Theta]^2} + \frac{2 \Theta [A_1 + y_2 + s_1] [\gamma - \theta + \pi \omega]}{[1 + \Theta]^2} + \frac{[\gamma - \theta + \pi \omega]^2}{[1 + \Theta]^2}
\]
We note that asset holdings at the end of the first period are
\[
A_1 = y_1 - c_1 - s_1
\]
\[
= \frac{\Phi_2 [1 + \Phi_2 \Theta] y_1 - (1 - P_1 \Theta) \Phi_2 y_2 + [2 + \Phi_2 + \Phi_2 \Phi_2 \Theta] \gamma + [1 + P_1] (-\theta + \pi \omega)}{[1 + P_1 \Theta] + \Phi_2 [1 + \Theta - P_1 \Theta]}
\]

Finally, we employ the optimal consumption and savings rules to find expected utility from the vantage point of late in the first period, following first-period economic activity but before the possibility of natural death. We assume this to be the point of opportunity for defection, which is after first-period economic activity but before an early death may occur.
\[
\mathcal{L}_2^0 (s_1, A_1) = P_1 [\pi B + \pi \omega s_1 - (1 - \pi) D]
\]
\[
+ \Phi_2 \left[ \theta c^0 - \frac{\Theta}{2} [c^0]^2 + (\gamma + \pi \omega) s_2 - \frac{1}{2} [s_2]^2 + [\pi B - (1 - \pi) D] \right]
\]
\[
= [\pi B - (1 - \pi) D] + P_1 \pi \omega s_1
\]
\[
+ \Phi_2 \left[ \theta + (\gamma + \pi \omega) \Theta - \frac{\Theta}{2} [A_1 + y_2 + s_1] \right] \frac{A_1 + y_2 + s_1}{1 + \Theta} + \frac{1}{2} \left( \gamma - \theta + \pi \omega \right)^2
\]

We see that expected utility is the expected value of immediate death and advance to the afterlife plus the expected value of survival, followed by economic and religious activity in the second period, and ultimately ending in natural death.

**Defection**  
We now consider the case of the defector. At the beginning of the first period, all conditions are the same as before. The optimal first-period consumption level again is
\[
c_1 = y_1 + \Phi_2 y_2 - \gamma - [\Phi_2 + P_1 P_2 (-\theta + \pi \omega)]
\]
\[
= \frac{[1 + \Phi_2 + \Phi_2 \Phi_2 \Theta] \gamma + P_1 [1 + \Phi_2] (-\theta + \pi \omega) + P_1 \Theta [y_1 + \Phi_2 y_2]}{1 + \Phi_2 + [\Phi_2 + P_1 P_2 \Theta]} \Phi_2
\]

and first period investment is
\[
s_1 = \frac{\gamma + P_1 [-\theta + \pi \omega + \Theta c^0]}{1 + \Phi_2 + [\Phi_2 + P_1 P_2] \Theta}
\]
\[
= [1 + \Phi_2 + \Phi_2 \Phi_2 \Theta] \gamma + P_1 [1 + \Phi_2] (-\theta + \pi \omega) + P_1 \Theta [y_1 + \Phi_2 y_2]
\]
\[
+ \Phi_2 \left[ \theta + (\gamma + \pi \omega) \Theta - \frac{\Theta}{2} [A_1 + y_2 + s_1] \right] \frac{A_1 + y_2 + s_1}{1 + \Theta} + \frac{1}{2} \left( \gamma - \theta + \pi \omega \right)^2
\]

We note that asset holdings following first-period consumption and investment also are the same as before.

At this point, things change. The agent unexpectedly is presented with the opportunity to defect and earn \( y' \) in additional income. The cost will be \( s_1 \), his stock of religious capital. If death should occur in the first period, then he has no opportunity to rebuild capital levels and thus receives no returns in the afterlife.

The second-period budget constraint, where first-period capital stock has been lost but reward for defection have been added, is
\[
A_1 + y_2 + y' \geq c_2 + s_2
\]
For now, we do not consider the possibility that the agent would lose economic capital or escape debt through defection, though these seem likely and could be incorporated in the analysis. The optimal balance between consumption and capital stock in the second period again is

\[ s_2 = \Theta c_2 + [\gamma - \theta + \pi \omega] \]

By employing the budget constraint and optimal consumption-capital stock balance, we find the optimal second-period consumption and investment levels for the defector:

\[
\begin{align*}
    c_2 &= \frac{A_1 + y_2 + y'}{1 + \Theta} - \frac{\gamma - \theta + \pi \omega}{1 + \Theta} \\
    s_2 &= \Theta \frac{A_1 + y_2 + y'}{1 + \Theta} + \frac{\gamma - \theta + \pi \omega}{1 + \Theta}
\end{align*}
\]

These rules look similar to those for the loyal conspirator, but capital has been lost and defection rewards added to available resources. The second-order terms may be written as

\[
\begin{align*}
    [c_2]^2 &= \frac{[A_1 + y_2 + y']^2}{[1 + \Theta]^2} - \frac{2 [A_1 + y_2 + y'] [\gamma - \theta + \pi \omega]}{[1 + \Theta]^2} + \frac{[\gamma - \theta + \pi \omega]^2}{[1 + \Theta]^2} \\
    [s_2]^2 &= \Theta \left[ \frac{A_1 + y_2 + y'}{1 + \Theta} \right]^2 + \frac{2 \Theta [A_1 + y_2 + y'] [\gamma - \theta + \pi \omega]}{[1 + \Theta]^2} + \frac{[\gamma - \theta + \pi \omega]^2}{[1 + \Theta]^2}
\end{align*}
\]

We now have the optimality conditions necessary to determine lifetime utility for the defector, from the vantage point of late in the first period.

\[
L_D^2 (s_1, A_1) = P_1 [\pi B - (1 - \pi) D] + \Phi_2 \left[ \theta c_2 - \frac{\Theta}{2} [c_2]^2 + \gamma s_2 - \frac{1}{2} [s_2]^2 + [\pi (B + \omega s_2) - (1 - \pi) D] \right] = [\pi B - (1 - \pi) D] + \Phi_2 \left[ \theta + (\gamma + \pi \omega) \Theta - \frac{\Theta}{2} [A_1 + y_2 + y'] \right] \frac{A_1 + y_2 + y'}{1 + \Theta} + \frac{1}{2} \left[ \gamma - \theta + \pi \omega \right]^2
\]

Note that if death should occur in the first period, the agent receives no rewards in the afterlife for sacrifices made in the first period. If the agent survives, then he may invest in the second period to rebuild his religious capital. Again, we assume that redemption is possible, at least to some degree, and that investment of ill-gotten wealth is equivalent to any other sacrifice.

The Choice to Defect We now determine whether the optimizing agent will choose loyalty or defection. We will denote as \( \Delta_{ij} \) the difference in utility between choice \( i \) and choice \( j \). We denote the decision to defect as \( D \), and we
again use 0 to denote the decision to avoid killing and remain loyal. We then find

$$\Delta \rho_0 = [\pi B - (1 - \pi) D]$$

$$+ \Phi_2 \left[ \left( \theta + (\gamma + \pi \omega) \Theta \right) - \frac{\Theta}{2} [A_1 + y_2 + y'] \right] \frac{[A_1 + y_2 + y']}{1 + \Theta} + \frac{1}{2} \left[ \frac{[\gamma - \theta + \pi \omega]^2}{1 + \Theta} \right]$$

$$- \left[ + \Phi_2 \left( [\theta + (\gamma + \pi \omega) \Theta] - \frac{\Theta}{2} [A_1 + y_2 + s_1] \right) \frac{[A_1 + y_2 + s_1]}{1 + \Theta} + \frac{1}{2} \left[ \frac{[\gamma - \theta + \pi \omega]^2}{1 + \Theta} \right] \right]$$

$$= -P_1 \pi \omega s_1 + \frac{\Phi_2}{1 + \Theta} \left( \theta + \Theta \left( \gamma + \pi \omega - \left( A_1 + y_2 + \frac{y' + s_1}{2} \right) \right) \right) \left( y' - s_1 \right)$$

Note that the sum of resource terms in parentheses will be positive, since expected solvency requires that first-period debt must be smaller than second-period income. If these resources are greater than the rewards for capital and the preference for consumption, then defection might require loss of utility even for large reward. On the other hand, if the term in brackets is positive, as it will be if religious capital is highly valued, then larger payoff makes defection more attractive, though this function is concave in rewards $y'$.

The decision to defect carries an immediate cost, which is the expected value of returns to first-period capital stock that were lost through defection, given the first-period mortality rate and the perceived likelihood of gaining entry to heaven. This may be offset, to some degree, by the expected value of survival through the second period with the benefits of consumption and new religious capital, followed by eternity with the returns earned through the new investment. The immediate loss of capital tends to make the agent more loyal, and agents with high capital stocks will be particularly reluctant to defect. Agents with little stock may be enticed more easily, especially if mortality rates are low.

### 4.2 Outside Options

Berman [2] and Berman and Laitin ([3] and [4]) describe how radical religious groups typically require sacrifice of education, earnings opportunities, and ties to the outside world. These sacrifices leave the group member with poor outside economic options. Because they are aware that the alternatives are few, members of radical groups have high defection constraints and strong allegiance to the community. If improved outside options were to lower the defection constraints, then terrorist groups might be left vulnerable. We employ the model to examine the implications of greater outside opportunities.

Consider an increase in the exogenous second-period income level, which is available whether or not the agent defects. We imagine that the anticipated second-period income level improves unexpectedly late in the first period, after the first-period consumption, investment, and savings decisions have been made. According to the Equation 11 and for given levels of religious capital, higher
second-period income will affect the willingness to defect according to

$$\frac{\partial \Delta_{D_0}}{\partial y_2} = -\Phi_2 \frac{\Theta}{1 + \Theta} (y' - s_1)$$

When reward for defection is low \((y' < s_1)\), then willingness to defect seems to increase with income, though the reward still might not be sufficient to entice defection. When rewards are high \((y' > s_1)\), then higher income leads to a decreased willingness to defect. By calculating the cross partial derivative, we find

$$\frac{\partial^2 \Delta_{D_0}}{\partial y_2 \partial y'} = -\Phi_2 \frac{\Theta}{1 + \Theta}$$

indicating that income and rewards may act as substitutes in the member’s willingness to defect.

The way in which we model outside options is important. In the derivations above, we considered increases to income whether or not the agent left the radical group. However, continued membership likely would limit any increase in income, since membership typically imposes restrictions on education and on earning potential. Thus, it might be better to consider improvements in outside options as an increase in rewards for defection. In this case, we find the effect of improved options as

$$\frac{\partial \Delta_{D_0}}{\partial y'} = -\Phi_2 \frac{\Theta}{2 (1 + \Theta)} (y' - s_1) + \Phi_2 \frac{\Theta}{1 + \Theta} \left[ \theta + \Theta \left( \gamma + \pi \omega - \left( A_1 + y_2 + \frac{y' + s_1}{2} \right) \right) \right]$$

$$= \frac{\Phi_2}{1 + \Theta} \left[ -\Theta (y' - s_1) + \left[ \theta + \Theta \left( \gamma + \pi \omega - \left( A_1 + y_2 + \frac{y' + s_1}{2} \right) \right) \right] \right]$$

$$= \frac{\Phi_2}{1 + \Theta} \left[ \theta + \Theta \left[ \gamma + \pi \omega - \left( A_1 + y_2 + y' \right) \right] \right]$$

In the next section, we will argue that strong preferences for religious capital is necessary for membership in the group. If this is true, then the derivative likely is positive, at least for small defection payoffs. (We see also that \(\Delta_{D_0}\) clearly is concave in defection payoff.) This implies that improved outside options, and bigger bribes, increase the willingness to defect. Willingness increases directly with survival rates, where survival and ample resources allow significant redemption.

### 4.3 The Selection of Martyrs

Berman and Laitin ([3] and [4]) present models of radical religious groups as clubs. Certainly not all radical groups support terror. Those that do seem particularly potent. These clubs of religious radicals screen potential members to eliminate candidates who might prove to be free riders. These free riders would put their own interests before their concern for the group. Once a group of committed radicals has been formed, then this group of individuals who survived the initial screening provides a pool of promising candidates for endeavors that
require still deeper commitment, such as terror operations and suicidal attacks. Even for conventional insurgency operations, loyalty is essential. Since it is one of few effective options, suicidal attacks are preferred for high-value, hardened targets, even though it requires the sacrifice of a valuable asset. We examine here the case of loyalty in the limiting case where one’s own life is sacrificed willingly.

We now consider how terror group leaders might examine agents in the model to identify individuals who would become committed members of radical groups and who then might be suitable candidates for terror missions. Those who place high value on martyrdom, both for its natural benefits (σ) and supernatural rewards (Σ), likely would be suitable candidates for suicide, but these characteristics are not observable or easily verifiable to group leaders. What may be observable is economic sacrifice and the accumulation of religious capital. Religious capital is prized both for earthly benefits (γ) and afterlife reward (ω). Great individual sacrifice might indicate both strong commitment to the community and strong desire for afterlife reward. These suggest that the individual would remain loyal and might imply similar preferences and beliefs for martyrdom. Leaders of the organization thus would seek individuals who have accumulated large amounts of religious capital, both for their willingness to support the group and possibly to sacrifice life itself. We derive the results in the following text.

We first consider the implications for defection of higher capital stocks alone, without direct consideration of beliefs and preferences. We now examine the case of the collaborator, but not the case of the martyr. Higher capital affects the appeal of defection according to

\[ \frac{\partial \Delta D_0}{\partial s_1} = -P_1 \pi \omega - \frac{1 + \Phi_2}{\omega} \left( \frac{y - s_1}{2} \right) - \frac{\Phi_2}{1 + \Theta} \left[ \theta + \Theta \left( \gamma + \pi \omega - \left( A_1 + y_2 + \frac{y' + s_1}{2} \right) \right) \right] \]

\[ = -\frac{P_1}{1 + \Theta} \Theta \pi \omega - \frac{\Phi_2}{1 + \Theta} \left[ \theta + \Theta \left( \gamma - (A_1 + y_2) \right) \right] + \frac{\Phi_2}{1 + \Theta} s_1 \]

Since solvency requires that \( y_2 > A_1 \), the sign of the derivative is not clear, even when stocks are small. Though it seems reasonable that higher levels of capital make the agent less likely to defect, we will need to push further to find support from the model for this supposition.

To find the effects on investment of preferences for capital in the afterlife, we return to our work on the single-agent model. We find that stronger preference, or anticipation of higher returns, leads to higher first-period capital accumulation.

\[ \frac{\partial s_1^0}{\partial \omega} = P_1 \pi \frac{1 + \Phi_2}{1 + \Phi_2 + [P_1 + \Phi_2 \Phi_2] \Theta} \]

We likewise see that stronger preference for capital in the natural world leads to greater accumulation.

\[ \frac{\partial s_1}{\partial \gamma} = \frac{1 + \Phi_2 + \Phi_2 \Phi_2 \Theta}{1 + \Phi_2 + [P_1 + \Phi_2 \Phi_2] \Theta} \]
High capital accumulation is a necessary, though not sufficient, condition for strong preference for religious capital. With knowledge of actual and potential income and the chosen distribution between consumption and investment, all which likely are observable, it seems that relatively high investment may be a sufficient condition for strong preference for religious capital.

We next calculate the effects of stronger preferences for capital in the afterlife on the tendency to defect. We include the effects on first-period investment of a marginal change in preference.

\[
\frac{\partial \Delta P_0}{\partial \omega} = -P_1\pi s_1 + \frac{\Phi_2\Theta}{1 + \Theta} \pi (y' - s_1)
\]

\[
-\frac{P_1\pi\omega}{\partial \omega} \frac{\partial s_1}{\partial \omega} - \frac{\Phi_2\Theta}{2(1 + \Theta)} (y' - s_1) \frac{\partial s_1}{\partial \omega}
\]

\[
-\frac{\Phi_2}{1 + \Theta} \left[ \theta + \Theta \left( \gamma + \pi\omega - \left( A_1 + y_2 + \frac{y' + s_1}{2} \right) \right) \right] \frac{\partial s_1}{\partial \omega}
\]

\[
= -P_1 s_1 - \frac{\Phi_2\Theta}{1 + \Theta} \frac{\Phi_2\Theta [P_1 + \Phi_2\Theta]}{1 + \Phi_2\Theta} s_1 + \frac{\Phi_2\Theta}{1 + \Theta} \left[ (1 + \Theta) \pi\omega - \frac{\Phi_2\Theta}{1 + \Theta} (\gamma - (A_1 + y_2)) \right] \frac{\partial s_1}{\partial \omega}
\]

We see that, except possibly in the case of large bribes or high second-period income, the model implies that higher preference for capital in the afterlife will lead to a decreased willingness to defect.

We next calculate the effects of stronger preferences for capital in the present world on the tendency to defect. We again include the effects on first-period investment of marginal change in preference.

\[
\frac{\partial \Delta P_0}{\partial \gamma} = -P_1\pi s_1 + \frac{\Phi_2\Theta}{1 + \Theta} \pi (y' - s_1) + \frac{\Phi_2}{1 + \Theta} \left[ \theta + \Theta \left( \gamma + \pi\omega - \left( A_1 + y_2 + \frac{y' + s_1}{2} \right) \right) \right] \frac{\partial s_1}{\partial \gamma}
\]

\[
= -P_1 s_1 - \frac{\Phi_2\Theta}{1 + \Theta} \frac{\Phi_2\Theta [P_1 + \Phi_2\Theta]}{1 + \Phi_2\Theta} s_1 + \frac{\Phi_2\Theta}{1 + \Theta} \left[ (1 + \Theta) \pi\omega - \frac{\Phi_2\Theta}{1 + \Theta} (\gamma - (A_1 + y_2)) \right] \frac{\partial s_1}{\partial \gamma}
\]

Once again, we see that, except possibly in the case of large bribes or high second-period income, the model implies that higher preference for capital in the natural life will lead to a decreased willingness to defect. Additional work to specify the equation only in terms of parameters may provide a clearer indication of the sign of this derivative.

The model then supports the idea that stronger natural preference for religious capital and the stronger anticipation of returns to capital in the afterlife will lead the agent both to accumulate more capital and to be more resistant to defection, with the possible exception of high expected income later in life leading to an increased willingness to defect. If religious capital stocks or investment is observable, then large holdings indicate a high defection constraint and may provide a strong signal of loyalty.
So far, we examined the case of the collaborator, but we did not consider the would-be martyr. We can continue this analysis to consider the implications of religious investment for martyrdom. We found earlier that anticipation of higher afterlife returns will make martyrdom more attractive relative to the case of loyalty without martyrdom.

\[
\frac{\partial \Delta_{20}}{\partial \omega} = -\lambda^2 [P_1 \pi + \Phi_2 \Phi_2] + \lambda^0 [P_1 + \Phi_2 \Phi_2] \pi
+ \Phi_2 (1 - \pi) (1 + \pi) \omega + \Phi_2 (1 - \pi) \gamma
\]

We also derived the effects of higher preference for capital in the natural world and found that greater preference for capital stock on earth also leads to greater tendency to choose martyrdom.

\[
\frac{\partial \Delta_{20}}{\partial \gamma} = \Phi_2 (1 - \pi) \left[ \frac{1 + \Phi_2 + P_1 P_1 \Theta}{1 + P_1 \Theta + \Phi_2 [1 + (1 - P_1) \Theta]} \right] \omega
\]

If we assemble these pieces, we see that greater preference for religious capital signals willingness to resist defection and remain deeply loyal to the community. This preference for capital leads to greater investment, and this in turn provides a visible signal to group leaders that the individual would make a good candidate for membership. Once the individual is accepted, the higher capital stock leaves the member less inclined to defect. This reluctance to betray the group is reinforced by the strong preference for earthly benefits of capital and anticipation of eternal reward. Finally, these same preferences make the individual more willing to sacrifice life itself. High investment levels thus signal both loyalty and willingness to die a martyr's death.

5 Conclusion

We began with a single agent model of a religious person who contemplates martyrdom. The model was simple and perhaps simplistic and incorporated a number of extreme views. Rather predictably, it produced a number of extreme predictions. Still, the model provides a framework for incorporating religious beliefs in a standard economic model, along with a choice set that includes the decision to end life. The model yields many results that seem obvious, such as the belief that anticipation of reward in the afterlife could motivate terror, though the importance of the religious beliefs remains in question. Many parts of the model remain vague and abstract, such as the expected natural rewards for terror. These can be expanded to produce a more sophisticated model.

We extended the analysis of individuals by using the single-agent model to construct a multi-agent model of terrorist groups. We saw that agents who have accumulated religious capital will be more loyal and resistant to attempts to lure defectors, and the agent will be more inclined to accept martyrdom.

The analytical work likely can be improved with alternative functional forms that may allow tidier derivations and avoid the unlikely implications of quadratic
utility functions. Interesting extensions can be pursued using numerical modeling techniques. In particular, we hope to tailor the model to specific religious beliefs and consider optimal anti-terror policies for various religious doctrines. The work will be improved through greater incorporation of the nonreligious motivations for terror as studied by Berman, Laitin, and others.

References


