A Theory of Congregational Giving

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Abstract

This paper proposes a model to explain a broad range of established empirical facts about giving and attendance rates in religious congregations. We treat the religious service collectively consumed by the congregation as a “participatory” public good, in the sense that while its quality increases in contributions, individual consumption varies by the amount of time devoted towards attendance. The model predicts that lower income individuals will be over-represented in religious congregations, with giving concentrated among higher income members. Inclusive doctrine is shown to increase membership but reduce average giving and attendance, while “tithing” requirements reduce membership and increase total giving.

Keywords: Public Goods, Voluntary Contributions, Charitable Sector, Congregations

JEL Codes: H41, L31, D70, C72

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1 Introduction

Over a third of the estimated $306.39 billion in US charitable giving went to religious organizations,\(^1\) making this sector the largest single recipient of contributions in 2007 (Giving USA, 2008). While a large economic literature has developed on charitable contributions, little attention has been paid to the unique set of individual incentives and institutional features that can affect giving in religious congregations. At the same time, a growing body of empirical work has uncovered patterns in membership, attendance and giving rates that vary widely across individuals and across congregations. This paper explains these patterns by extending the traditional economic model of voluntary contributions to allow for additional individual choices of attendance and congregational membership. Also consistent with empirical findings, the model shows how these individual patterns are affected by the doctrine embodied in a congregation’s religious service, as well as by institutionalized practices of giving such as tithing.

Among the factors that sustain the continuing operations and survival of the typical congregation, none is more important than the voluntary financial contributions from its members. Using a nationwide representative sample of religious congregations in the US, Chaves (2004) finds that “(t)hree quarters of congregations receive at least 90 percent of their income from individual donations”. While there are presumably many reasons why people may contribute financially to these groups, it is hard to deny that most congregants share a realization that their financial contributions tangibly and positively affect the regular operations of the religious service that the congregation provides. We choose to focus on this motivation as a starting point for explaining the observed variation across individual giving decisions within

\(^1\)Giving to “religious organizations” does not include separately incorporated faith-based organizations that provide services such as education or healthcare and is predominantly giving from individuals or households to religious congregations (Giving USA, 2008). Congregations are groups of religious believers that meet together on a regular basis as a collective expression of their faith. There are over 330,000 such congregations in the United States, with roughly 300,000 of these being Protestant and other Christian churches, 22,000 being Catholic and Orthodox churches and 12,000 non-Christian congregations (Estimates from Hadaway and Marler, 2005).
congregations.

Building on the basic framework of voluntary contributions (Bergstrom, Blume & Varian, 1986), the model presented here treats the religious service collectively consumed by congregation members as a type of participatory public good. In many ways, the religious service collectively consumed by a congregation has the basic features of a public good. Although individuals may differ in their tastes for the service, it is non-rival in the sense that one congregation member’s involvement in the service does not preclude consumption by others. Moreover, the quality of the religious service (i.e., the quality and furnishing of religious facilities; maintenance of musical instruments and sound equipment; the number of salaried clergy members; the existence of children’s programs and so forth) depends largely on the amount of voluntary contributions from members of the congregation.

A point of departure in the model is that while the public value of the service is positively affected by contributions, consumption by members is allowed to vary by the amount of time they devote towards attendance. In this sense, even if an individual chooses to contribute nothing financially to the congregation, they must sacrifice time in the form of attendance to derive any benefit from the collective service it produces. This changes the nature of “free riding” so that in addition to characterizing a set of core contributors by income and tastes, the model also provides results on how attendance decisions vary by individual characteristics as well as the profile of giving decisions in the congregation.

Before dealing specifically with the results of the model, it should be noted that this paper adds to a small but growing economic literature on the theory of religious behavior. Azzi and Ehrenberg (1975) and later Sullivan (1985) examine religious giving decisions from the perspective of an individual isolated decision maker. The former treats religious giving as a type of savings towards consumption in an “afterlife” period and, although empirical support for the model is mixed (Iannaccone, 1998), their work stands as the first modern economic analysis of religious behavior. While the framework in these papers is useful for examining

\[\text{Prior to Azzi and Ehrenberg (1975), the only substantive economic analysis of religious behaviour is contained in an often overlooked discussion in Adam Smith’s *The Wealth of Nations* (1776). In this section (Book V, Chapter I, Article III), Smith discusses the impact of market forces on the growth and success of}\

\]
several life-cycle aspects of religious giving, they stop short of a more general theory of “congregations” because the analysis is centered on the decisions of an isolated individual or household and not on how those decisions interact and change with the decisions of other congregation members.³

More closely related to the work here, several papers in the economics literature model religious organizations as “clubs” (see Buchanan (1965); Cornes & Sandler (1984)). Building on this framework, Iannaccone’s (1992) seminal paper applies the theory of clubs to analyze the potential efficiency of seemingly wasteful rituals in religious sects and cults. Models in this vein focus on positive crowding effects to participation in these groups, and show that consumption sacrifices can increase participation when the group can monitor and restrict non-group behavior. In contrast, the work here shifts attention away from crowding effects in overall participation, and as explained next we focus on the interplay between the collective role of giving and the consumption role of attendance within and across congregations.

1.1 Giving Within and Across Congregations

A distinctive feature of the distribution of contributions within congregations is that it is typically highly “skewed”. That is, the majority of the income that a congregation receives comes from a core set of contributors with relatively high income (Hoge, 1994). In simulations where individuals contribute a fixed share of income, with this share increasing in a religious taste parameter, Iannaccone (1997) shows that skewness in giving can occur even when income and tastes are independently distributed. In the model presented here, we extend this result to show that when giving decisions are not made independently, but are determined jointly in the congregation, that a groups of core contributors and non-contributors naturally

³While, admittedly, some facets of religious behaviour may be entirely personal, we argue that the most interesting features of behaviour in congregations derive from their “collective” nature. Namely, how the giving and attendance of any congregation member are influenced by the characteristics and decisions of other congregation members.
arise.\textsuperscript{4} That is, when individuals take into consideration the contributing behavior of other members, giving is always concentrated among a contributing set with relatively high income and tastes.

Studies on the attendance decisions of individuals in congregations typically find a negative relationship between income and attendance (Lipford & Tollison, 2003) although some find this relationship to be relatively weak (Iannaccone, 1997). The model here serves to clarify these results by demonstrating that attendance rates will differ and, perhaps more subtly, that a different functional relationship between income and attendance will exist for contributors and non-contributors. Intuitively, non-contributors spend all of their disposable income on private consumption, and thus a small increase in their income increases the opportunity cost of church attendance if private consumption and leisure are complementary activities. Thus the model predicts a stronger negative relationship between income and attendance for non-contributors than for contributors who, in contrast, divide any increase in income between private consumption and increased giving.

Across congregations, Finke, Bahr & Scheitle (2006) examine the effect of strictness or exclusivity of doctrine on observed rates of giving. Using surveys of congregation leadership, measures of exclusivity constructed from how “literally” religious texts are interpreted as well as the amount of structure placed on the beliefs taught in religious services, and these measures are found to be positively related to giving rates.\textsuperscript{5} To account for these effects, we consider an environment in which the heterogeneity in tastes for the religious service is determined in part by the beliefs or ideology of individuals (assumed to be fixed), and in part by how much the service or “message” of the congregation appeals to these different sets of beliefs. In this framework, we are able to analyze how the patterns of membership, attendance and giving change between congregations with exclusive doctrine (those that appeal strongly to a narrow set of beliefs) and those with inclusive doctrine (those that

\textsuperscript{4}In his discussion on skewness in giving, Hoge (1994) notes that the existence of core groups and “periphery” is found in “all sociological studies of congregations”.

\textsuperscript{5}The finding that strict/exclusive doctrine correlates with high levels of giving and attendance is common in studies across congregations. See Olson & Perl (2001) and Lunn, Klay & Douglass (2001) for other examples.
appeal broadly to individuals with different beliefs). The intuition is that exclusivity in doctrine increases heterogeneity in tastes for the religious service by increasing its value to some types of individuals while reducing it for others. Thus the model predicts that while inclusive congregations will have larger memberships, exclusive congregations will have higher rates of giving and attendance. Moreover, since inclusive congregations do not appeal strongly to any one type of individual, tastes for the religious service are in general lower, resulting in giving being concentrated among those individuals with the highest levels of income.

Although giving in many congregations takes place as simple “passing the plate”, some have institutionalized practices of giving through tithing. In studies across denominations, the practice of tithing is found to increase giving (see for example Forbes and Zampelli (1997) and the references therein). To understand the effects of tithing on a congregation we focus on the strongest form of a tithing requirement, by allowing individuals to be excluded from membership if they fail to contribute a fixed proportion of their income. Since individuals must contribute a certain amount to join, the first effect of this requirement is that it creates a type of coordination problem in membership decisions. That is, individuals may wish to pay the tithe and join the congregation only when others do so as well. This leads to an interesting multiplicity of equilibria, not present in the model without tithing, with different sizes of memberships depending individual beliefs about how many others will join. Consistent with the empirical evidence, the model predicts that while tithing may reduce membership size, a small tithing requirement will encourage some members to give more, and will in fact always increase total giving in the congregation.

The outline of the paper is as follows. Section 2 describes the basic model and focuses on giving, membership and attendance decisions within a congregation. Sections 3 and 4 show how these decisions are affected by doctrine and tithing requirements, respectively, and section 5 concludes. Proofs are found in the Appendix unless stated otherwise.
2 A Model of Congregational Giving

Consider a model in which there is a finite set of individuals $N$ that are potential members of a religious congregation. Each individual consumes an amount $x_i$ of a private good and donates an amount $d_i \geq 0$ to the congregation. The congregation provides a non-rival religious service to those attending, and the quality of this service is increasing in the total amount of donations. Thus we model the religious service as a public good with quality given by $D = \sum_i d_i$. Each individual is endowed with an amount of income $w_i$ that can be used to purchase the numeraire private good or can be donated to the congregation.

In addition to income, individuals are endowed\(^7\) with a divisible amount of “disposable time” that can be consumed as leisure, $t_i$, or used to attend the religious service, $a_i$. We normalize this amount of time to 1 unit. Thus the constraints for individual $i$ are:

$$x_i + d_i \leq w_i \quad \text{and} \quad t_i + a_i \leq 1$$

where the income levels of each individual, $w_i \in \{w_1, \ldots, w_n\}$, are all strictly positive. Individuals have preferences over private consumption $x_i$, leisure $t_i$, and religious service consumption $(a_i, D)$ represented by the utility function:

$$U(t_i, x_i, a_i, D; \beta_i) = v(t_i, x_i) + m(a_i, D; \beta_i)$$

where $\beta_i$ is a preference parameter reflecting individual $i$’s taste for the religious service. We assume that $v$ and $m$ are increasing, twice continuously differentiable, and strictly concave in $(t_i, x_i)$ and $(a_i, D, \beta_i)$, respectively. We model leisure and private consumption as complements and assume that for all $i$, $v_{tx} = \partial^2 v / \partial t_i \partial x_i > 0$ with $\lim_{t \to 0} v_t = \infty$ and $\lim_{x \to 0} v_x = \infty$. We also make the following assumptions on $m$:

\textbf{A1:} $m_{D\beta} = \partial^2 m / \partial D \partial \beta_i > 0$ and $m_{a\beta} = \partial^2 m / \partial a_i \partial \beta_i \geq 0$.

\(^6\)We postpone “excludability” issues until a later section, and assume for the time being that the religious service is also non-excludable.

\(^7\)Note that this formulation assumes that individuals supply labour inelastically and abstracts from labour-leisure tradeoffs. This is done in part for simplicity, but also to focus on what we feel is the more important time use margin for congregation members. Namely, the tradeoff between leisure and religious attendance.
\textbf{A2:} \( m_{aD} = \partial^2 m / \partial a_i \partial D > 0 \) and for all \( \beta_i \) and \( D \geq 0 \), \( m(0, D; \beta) = 0 \).

Assumption A1 conceptually defines the parameter \( \beta_i \). In reality, individuals differ in the strength of their religious conviction as well as how closely aligned their beliefs may be to those collectively expressed by a particular congregation. These differences are modeled via the heterogeneity in the \( \beta_i \)'s of the congregation members, with higher \( \beta_i \)'s being associated with higher marginal utilities of religious service consumption.

The second assumption above clarifies the role of attendance for individual congregation members and represents a point of departure from models of public goods. Assumption A2 states that in order to receive any benefit from the religious service the individual must spend at least some time attending, and that the marginal utility of attendance is higher when religious service quality is higher.\(^8\) Note that unlike a typical “club good”, we do not assume that higher average attendance rates degrade the value of the religious service. However, the attendance decisions of individuals are linked through their response to the amount of donations in the congregation. As demonstrated below, this complementarity between attendance and total giving will lead to testable patterns in the donation and attendance rates within the congregation that vary based on the distribution of income among members.

\section*{2.1 Timing and Equilibrium}

We use the above framework to analyze the attendance and giving decisions of all members of the congregation. Individuals choose \((t_i, x_i, a_i, d_i)\) simultaneously and we define a Nash equilibrium of the game to be a vector of choices, \((t_i^*, x_i^*, a_i^*, d_i^*)_{i \in N}\), such that for all \(i\), \((t_i^*, x_i^*, a_i^*, d_i^*)\) solves:

\[
\max_{\{t_i, x_i, a_i, d_i\}} U(t_i, x_i, a_i, D; \beta_i) = v(t_i, x_i) + m(a_i, d_i + \sum_{j \neq i} d_j^*; \beta_i) \]

subject to \(x_i + d_i = w_i\)

\[
t_i + a_i = 1
\]

\(^8\)It is in this sense that the religious service is a self-excludable or “participatory” public good. That is, even those congregation members who decided not to contribute must still sacrifice leisure in order to get any benefit from the service.
Following standard arguments, we use the fact that any positive contributor \( i \) effectively chooses the level of total donations taking into account what others give in equilibrium, \( D_{-i} \). Thus we can rewrite the problem of individual \( i \) as:

\[
\max_{\{t_i, x_i, a_i, D\}} U(t_i, x_i, a_i, D; \beta_i) = v(t_i, x_i) + m(a_i, D; \beta_i)
\]

\[
\text{s.t.} \quad x_i + D \leq w_i + D_{-i}
\]

\[
t_i + a_i \leq 1
\]

with the additional restriction that \( D \geq D_{-i} \). Ignoring this last restriction and solving the optimization problem in (1) for individual \( i \) yields the Marshallian demand correspondences

\[
x_i^* = x_i(w_i + D_{-i}), \quad D_i^* = D_i(w_i + D_{-i}), \quad t_i^* = t_i(w_i + D_{-i}), \quad \text{and} \quad a_i^* = a_i(w_i + D_{-i}).
\]

Since the assumptions on \( v(\cdot) \) and \( m(\cdot) \) guarantee that \( U \) is strictly quasi-concave, these correspondences are all single valued functions of \( (w_i + D_{-i}) \equiv W_i \). Thus reintroducing the restriction that \( d_i \geq 0 \) by defining

\[
\delta_i(w_i, D_{-i}) = \max \{D_i(W_i) - D_{-i}, 0\}
\]

we have the following proposition.

**Proposition 1.** A Nash equilibrium exists for the congregation. Moreover, if \( x \) and \( D \) are normal goods for all individuals, the equilibrium is unique.

In what follows, we will maintain the normality assumption from Proposition 1 and consider the case where the equilibrium in the congregation is unique. To characterize this equilibrium, we first consider a simplified version of the model in which individuals differ only in their levels of income. As shown below, the predictions about individual behavior in this setting are stark but provide some basic intuition that will be useful in explaining the predictions in the more general setting of later sections.

### 2.2 Model with Identical Preferences

Consider the case where all individuals have identical preferences (ie. \( \forall i \beta_i = \beta \)) and differ only in their income levels \( w_i \in \{w_1, \ldots, w_N\} \). As discussed below, this simplified version
of the model predicts that members with higher income will make up the equilibrium set of positive contributors, while lower income (non-contributing) members will have higher rates of attendance. To begin, note that in equilibrium, contributors are those individuals who demand a higher level of quality, $D$, than what is provided from the contributions of others. That is, the following inequalities must hold for contributors and non-contributors:\footnote{Note that the lack of subscript on the demand function $D^*$ follows from the assumption here of identical preferences.}

\[
\begin{align*}
(\forall i \in C) \quad & D^*(w_i + D^*_i) > D^*_i \\
(\forall i \notin C) \quad & D^*(w_i + D^*_i) \leq D^*_i
\end{align*}
\]

These inequalities, together with the maintained assumption that $D^*$ is increasing lead to the following proposition.\footnote{To be precise, we exploit the fact that the marginal propensity to consume $D$ is between 0 and 1. The normality of $x$ and $D$ are sufficient for this to be true.}

**Proposition 2.** With identical preferences, there exists a unique $\hat{w}$ such that

\[
(\forall i) \quad d^*_i = \begin{cases} 
  w_i - \hat{w} & \text{if } w_i > \hat{w} \\
  0 & \text{if } w_i \leq \hat{w}
\end{cases}
\]

So the equilibrium with identical preferences is characterized by a unique threshold income $\hat{w}$. Individuals with income below $\hat{w}$ choose not to contribute and those with $w_i > \hat{w}$ contribute all their excess income $w_i - \hat{w}$. Note also that among the contributing set $C$ that the choices $(t^*, x^*, a^*)$ are all identical as the following proposition shows.

**Proposition 3.** With identical preferences, all contributors choose the same level of private consumption, leisure and attendance. That is, $(\forall i \in C), (t^*_i, x^*_i, a^*_i) = (t^*, x^*, a^*)$

**Proof:** Note that $(\forall i \in C)$ $w_i + D^*_i = w_i + D - d^*_i = w_i + D - (w_i - \hat{w}) = D + \hat{w}$. So

\[
\begin{align*}
  x^*_i &= x^*(w_i + D^*_i) = x^*(D + \hat{w}) = x^* \\
  t^*_i &= t^*(w_i + D^*_i) = t^*(D + \hat{w}) = t^* \\
  a^*_i &= a^*(w_i + D^*_i) = a^*(D + \hat{w}) = a^*
\end{align*}
\]

Although the consumption levels are the same among the contributing set, this is not so for non-contributors. Patterns of attendance among non-contributors depend non-trivially
on income levels as the next propositions shows. That is, non-contributors with the highest 
rates of attendance are those with the lowest levels of income, and this attendance rate drops 
as income rises.

**Proposition 4.** Among the set of non-contributors \((i \notin C)\), the optimal choices \((\tilde{t}_i, \tilde{x}_i, \tilde{a}_i)\) 
are such that

\[
(\forall i \notin C) \quad \tilde{t}_i \leq t^*, \quad \tilde{x}_i \leq x^*, \quad \text{and} \quad \tilde{a}_i \geq a^*
\]

with strict inequalities if \(w_i < \hat{w}\). Moreover, for all non-contributors, attendance is a mono-
tonically decreasing function of income.

To summarize, the equilibrium predictions of the model with identical preferences are as 
follows. The set of contributors are those with relatively high income, and these contributors 
donate all excess income above an endogenously determined threshold. Non-contributors are 
those with relatively low income and their attendance rates are negatively related to income. 
However, the prediction that all contributors choose identical levels of attendance and private 
consumption is particularly stark. In the next section, we allow for heterogeneity in tastes 
for the religious service and a show that richer set of predictions about giving and attendance 
patterns results.

### 2.3 Model with Preference Heterogeneity

Consider now the case in which individuals differ in their income levels, \(w_i\), as well as in their 
tastes for the religious service, \(\beta_i\). We impose no special conditions on the correlation between 
income and religious preference and assume only that for all \(i\): \(w_i \in (0, \bar{w}]\), \(\beta_i \in (0, \bar{\beta}]\) where 
\(\bar{w}\) and \(\bar{\beta}\) are finite. We will refer to the pair \((\beta_i, w_i)\) as individual \(i\)’s “type”, and assume that 
the distribution of types for the set of potential members is fixed and common knowledge.

**Proposition 5.** There exists a downward sloping locus of types in the \((\beta, w)\) plane that 
partitions the set of congregation members into contributors and non-contributors.

This locus of types divides the congregation with all contributors falling to the “northeast” 
(relatively higher \(w_i\) or \(\beta_i\)) and non-contributors having lower income or religious preference.
The intuition behind this result lies in the fact that the desired level of donations for each individual are increasing in both income and tastes for the religious service. Thus, if we were to perform the conceptual experiment of increasing the taste parameter $\beta_i$ for some individual $i$, this increase must be met with a corresponding decrease in income to keep their desired donation level the same. Given the equilibrium level of donations, focusing on those individuals that are exactly on the margin between giving some positive amount and giving nothing generates the locus of types characterized by the preceding proposition.

Given this partition, the following proposition characterizes giving and attendance rates for the congregation members by their types $(\beta_i, w_i)$.

**Proposition 6.** The following are true for positive contributors in the congregation:

1. Both giving and attendance are increasing in $\beta_i$ (holding income constant).
2. Giving increases with income, but attendance may increase or decrease.

and for non-contributors:

3. Attendance is increasing in $\beta_i$ and decreasing in income.

Notice that a stronger relationship exists between income and attendance for non-contributors than for contributors. Intuitively, this arises because non-contributors spend all of their disposable income on private consumption, and thus a small increase in their income increases the opportunity cost of attendance. This is in contrast to contributors who divide any increase in income between private consumption and increased giving. As such, the predicted relationship between income and attendance among contributors is weaker, and ultimately depends on the relative strengths of the complementarities between private consumption and leisure, and service quality and attendance.

In this section we do not consider any institutional features in the congregation that would exclude individuals from membership. As such, we define membership as the set of individuals that choose any positive level of attendance. Using this definition, the following proposition characterizes a separate locus of types in $(\beta, w)$ space that divides individuals into members and non-members.
**Proposition 7.** Among non-contributors, there exists an upward sloping locus of types in the \((\beta, w)\) plane that separates members from non members.

The above provides a possible explanation for why lower income groups tend to be over-represented in religious congregations. The negative relationship between income and attendance implies that among the individuals with lower tastes for the religious service, those with high income will choose not to attend at all. Moreover, when tastes for the religious service are low, non-contributing individuals with low levels of income (and thus low private consumption) may choose to attend since they have a smaller opportunity cost of devoting time towards attendance. Thus for any given distribution of income levels and tastes, the result above suggests that the observed distribution of income levels in the congregation will be more “bottom heavy” than the original population. We conclude this section with the following figure depicting the division of individuals into groups of members and non-members as well as contributors and non-contributors.

**Figure 1: Contribution and Membership Decisions**
3 Doctrine and Giving Rates

In the previous sections, we have maintained the assumption that the distribution of the $\beta_i$'s is fixed and exogenous. Consider now the case where the parameter $\beta_i$ for individual $i$ is higher when the religious service produced by the congregation is, in a sense, more closely aligned with his/her beliefs. That is, the heterogeneity in tastes for the religious service is determined in part by the beliefs or ideology of individuals (assumed to be fixed), and in part by how much the service or “message” of the congregation appeals to these different sets of dogmatic beliefs. The aim of this section is to characterize the difference in giving and attendance rates in congregations with “exclusive” doctrine (i.e. those that appeal strongly to only one set of individual beliefs) and those with “inclusive” doctrine (those that appeal broadly to more than one set of beliefs).\footnote{Measures of the exclusivity of doctrine have been constructed by empirical researchers studying the effects of doctrine on giving rates (For an example see Finke, Bahr & Scheitle (2006)). These measures are constructed from surveys of congregational leaders using responses to questions such as how “literally” religious texts are interpreted. Of course, two congregations with very different doctrine could both claim to interpret religious texts “literally”, but these responses allow for a measure of how accepting different doctrines are of a variety of different belief structures.}

To formalize the concept of inclusiveness, we assume that there are two groups of individuals that make up the set of potential congregation members, and that these groups differ in terms of their dogmatic beliefs. That is, the groups are located at either extreme of a doctrinal spectrum $[0,1]$. We refer to the location at point $k \in \{0,1\}$ as an individual’s type and assume, for clarity, that there are $n$ individuals of each type $k$ and that the distribution of income levels $\{w_1,...,w_n\}$ is the same for each type.\footnote{It is worth noting that these assumptions are made for convenience and that results in this section are generalizable to $l$ different types located at the extreme points of an $l$-dimensional unit simplex with different income distributions for each type.} The congregation, in terms of its doctrine, is located at point $\alpha \in [0,1]$ and we assume that individuals of type $k$, value the religious service it provides more if $\alpha$ is closer to $k$. We model this by assuming for each type that
\[(\forall i) \beta_i^k = \begin{cases} 
1 - \alpha & \text{if } k = 0 \\
\alpha & \text{if } k = 1 
\end{cases}\]

In this sense, we refer to a congregation’s doctrine as **exclusive** if it is located at either extreme \(\alpha \in \{0, 1\}\) and as **inclusive** if it appeals to both types, \(\alpha \in (0, 1)\). All other aspects of the model remain the same, but for concreteness we make the following preference assumptions

**A4:** For all \(w \in \{w_1, \ldots, w_n\}\) and \(D \geq 0\), \(\beta_i^k = 0\) implies \(v_i(1, w) > m_a(0, D; 0)\).

**A5:** For all \(\alpha \in [0, 1]\), there exists some \(i\) with income \(w_i\) such that \(D^*(w_i; \beta_i^k) > 0\).

Assumption A4 is made to sharpen what is meant by an exclusive doctrine. Namely, if a doctrine appeals exclusively to one type, then the marginal value of leisure is always higher than marginal value of attendance for the other type. Likewise, assumption A5 justifies the use of the term “inclusivity”, by restricting attention away from the case where interior values of \(\alpha\) destroy the value of the religious service for every individual.\(^{13}\) With these assumptions in place, we are now able to characterize how giving and attendance rates vary with the doctrinal “location” of the congregation. Note that we do not consider here the problem of how a congregation sets, or should set, its doctrine. Rather, in what follows we focus only on the positive analysis of how we expect individuals of different types to behave for any exogenously given doctrinal position \(\alpha\).\(^{14}\)

### 3.1 Equilibrium Characterization

To characterize equilibrium rates of giving and attendance across inclusive and exclusive congregations, we begin by noting that Proposition 1 holds for any distribution of \(\beta_i^k\)’s for

\(^{13}\)One could also rewrite this assumption as a primitive assumption on preferences as follows. First define \(\hat{a}_i(x, D) = \arg\max_{a \in [0, 1]} \{v(1 - a, x) + m(a, D; \beta_i^k) \mid (x, D) \in \mathbb{R}^2_+\}\) as the optimal choice of time use for any level of private consumption and total donations. Then A5 can be written as: For all \(\alpha \in [0, 1]\), there exists some \(i\) with income \(w_i\) such that \(m_D(\hat{a}_i(w_i, 0), 0; \beta_i^k) > v_x(1 - \hat{a}(w_i, 0), w_i)\).

\(^{14}\)For many congregations, doctrine is set in broad terms at the denominational level and not by individual congregations. For a model endogenizing these decisions at a denominational level see Rosborough (2009).
potential congregation members. Thus for any given $\alpha$, we have a unique equilibrium, with total donations $D(\alpha)$ and contributing set $C(\alpha)$. We consider first an equilibrium with $\alpha = 1/2$. Notice that this corresponds to the case where all individuals have identical preferences, and the equilibrium profile of contributors can be characterized by a unique threshold income level $\hat{w}$.

A natural question then is how the contributions from each type change as $\alpha$ moves away from 1/2. To answer this question, we define $C^k(\alpha)$ as the equilibrium set of contributors of type $k$ for doctrinal position $\alpha$, and make use of the following lemma. Lemma 1 shows, in the region where some of the individuals of each type contribute, that the donations of type 0 individuals are decreasing in $\alpha$ and the donations of type 1 individuals is increasing in $\alpha$.

**Lemma 1.** If $\alpha \in (0,1)$ is such that $C^k(\alpha)$ is non-empty for $k \in \{0,1\}$, then changes in $\alpha$ that don’t affect the set of contributors satisfy:

$$(\forall i \in C^0) \frac{\partial d^0_i}{\partial \alpha} < 0 \quad \text{and} \quad (\forall i \in C^1) \frac{\partial d^1_i}{\partial \alpha} > 0.$$ 

This result demonstrates that individuals of either type $k \in \{0,1\}$ respond to changes in $\alpha$ by increasing their donations when $\alpha$ is closer to $k$. The main idea in the proof is that the threshold income levels, $\hat{w}^k$, that divide the set of individuals of a given type into contributors and non-contributors decrease as the congregation’s doctrine is more closely aligned with that type. While the results above hold when marginal changes in $\alpha$ don’t change the equilibrium set of contributors, the following lemma demonstrates that these thresholds are continuous in $\alpha$, even when the set of contributors changes.

**Lemma 2.** $\hat{w}^k$ is continuous in $\alpha$ for $k \in \{0,1\}$.

**Proof:** Note that by assumption $D^*(w_i; \beta^k)$ is continuous in $\alpha$ for all $i$. Thus $D = \sum_i \max\{D^* - D_{-i}, 0\}$ is continuous in $\alpha$, and so $\hat{w}^k \equiv \phi(D(\alpha); \beta^k) - D(\alpha)$ is also continuous in $\alpha$. □

The intuition behind the above lemma is that even when a small change in $\alpha$ changes the contributing set, the set of “marginal contributors” must be giving exactly zero. The result is that the thresholds, $\hat{w}^k$, do not make discontinuous “jumps” as the contributing set changes.
Using this fact, we can now show that as the congregation becomes more exclusive, the set of contributors will be composed of only one type.

**Lemma 3.** There exists an $\alpha \in (1/2, 1)$ such that for all $\alpha > \alpha$ only type 1’s contribute, and there exists an $\alpha \in (0, 1/2)$ such that for all $\alpha < \alpha$ only type 0’s contribute.

The proof of the above lemma defines $\bar{w}$ to be the highest income level among individuals of type $k$ and shows that the threshold income level for contributors of that type surpasses $\bar{w}$ as $\alpha$ diverges from $k$. The results of lemmas 1-3 are summarized in the following figure.

**Figure 2: Doctrine and Giving Thresholds**

Using the above results we are now able to show the following proposition comparing behavior across congregations. For clarity of comparison, the proposition treats inclusive congregations as those with $\alpha = 1/2$. 

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Proposition 8. The following are true comparing exclusive congregations ($\alpha \in \{0, 1\}$) and inclusive congregations ($\alpha = 1/2$):

1. Membership is smaller in exclusive congregations.
2. Average giving is higher in exclusive congregations.
3. The average income of contributors is higher in inclusive congregations.
4. If an inclusive and exclusive congregation have the same level of total donations, then average attendance will be higher in the exclusive congregation.

4 Tithing and Membership Decisions

Although giving commonly takes place simply as “passing the plate” in many congregations, some denominations including the Church of Jesus Christ of Latter Day Saints (or the Mormon Church) have institutionalized practices of giving through tithing. Tithing stipulates that members of a congregation give a certain percentage of their income to the congregation.\textsuperscript{15} In this section we consider a mandatory tithing requirement as a prerequisite for congregation membership and analyze the impacts on membership and total giving. We find that although tithing may reduce membership, implementing a small tithing requirement always increases total giving for the congregation.

To see this, we consider a congregation that implements a policy that each potential member $i \in N$ donate a minimum of $\tau w_i$ in order to attend its religious service, where $\tau \in (0, 1)$. Since membership decisions involve more than simply choosing $a^*_i > 0$ as in the basic model, we now model the decision process of individuals as a two stage game. In the first stage, individuals choose whether or not to pay the tithing requirement and join the congregation, and in the second stage members choose how much they will contribute (if at all) above the tithe. Focussing first on the giving stage, note that if $M \subseteq N$ members join the congregation, total donations are given by

\textsuperscript{15}Strictly speaking, the word “tithing” (deriving from the word “tenth”) applies to the practice that members give 10% of their income to the church. However, the analysis in this section applies generally to any mandatory income-proportional giving requirement.
\[ D = \sum_{i \in M} g_i + \sum_{i \in M} \tau w_i \]

where the \( g_i \) should be interpreted as donations \textit{over and above} the tithing requirement by individual \( i \). Defining \( G_{-i} \equiv G - g_i \) as the total amount of donations (over and above tithing) by all individuals other than \( i \), we may again write the problem of an individual member as

\[
\max_{\{t_i, x_i, a_i, D\}} \quad U(t_i, x_i, a_i, D; \beta_i) = v(t_i, x_i) + m(a_i, D; \beta_i) \\
\text{s.t.} \quad x_i + D \leq (1 - \tau)w_i + G_{-i} + \sum_{j \in M} \tau w_j \\
\]  

(2)

with the additional restriction that \( D \geq G_{-i} + \sum_{j \in M} \tau w_j \). As in the basic model, ignoring this last restriction and solving the optimization problem in (2) for individual \( i \) yields the Marshallian demand function

\[ D_i^* = D_i((1 - \tau)w_i + G_{-i} + \sum_{j \in M} \tau w_j). \]

Notice that allowing for a tithing requirement is a “continuous” extension of the basic model and all analysis is left unchanged if we set \( \tau = 0 \). Accordingly, we can define the functions

\[ \delta_i(w_i, G_{-i}, \tau) = \max\{D_i^* - G_{-i} - \sum_{j \in M} \tau w_j, \ \tau w_i\} \]

for all \( i \in M \) as in the original model. Note that for a given \( M \), existence and uniqueness of equilibrium follow from a simple generalization of Proposition 1. Thus, for each profile of membership decisions among the \( N \) individuals we know that there is a unique level of total donations and that the set of members is divided uniquely into a set of “tithers” and those that give over and above the tithe. Thus we are able to compare an original contribution equilibrium to a new equilibrium with membership \( M \) after tithing is imposed. We show in the following proposition that if the tithing requirement does not discourage all original non-contributors from attending, then total donations must increase.
Proposition 9. Consider an equilibrium with a set \( N^0 \) of members and \( \tau = 0 \) in which total donations are \( D \) and the contributing set is \( C \); and an equilibrium after a tithing requirement \( \tau > 0 \) is implemented, with membership \( M \), total contributions (including tithing) \( D' = \sum_{i \in M} g_i + \tau \sum_{i \in M} w_i \).

Then if \( C \subseteq M \), we must have \( D' > D \).

4.1 Equilibrium with Tithing

The result above gives some characterization to total giving for a given membership profile. In effect, these giving decisions are taking place in a subgame associated with a specific profile of membership decisions by all individuals. In order to characterize the equilibrium membership decisions in the first stage, we will make use of the fact that for any profile of these membership decisions there is a unique equilibrium in the contributions subgame that follows it. We will also need the following lemma about how total contributions in the subgame equilibria change with an increase or decrease in membership. This lemma states that if there is an increase in membership, then total contributions (ie. the sum of tithes and gifts) must increase.

Lemma 4. Given a tithe \( \tau \), consider two contributions subgame equilibria with memberships \( M_0 \) and \( M_1 \). If \( M_0 \subset M_1 \) then total contributions in the equilibria satisfy \( D_0 < D_1 \).

Using Lemma 4, we are able to construct a general existence result for equilibria of the entire game. The equilibrium concept that we will adopt is that of subgame perfection. That is, given the fact that for each membership there is a unique equilibrium at the contribution stage, an equilibrium of the entire game occurs when no individual wishes to deviate from their membership decisions in the first stage.

We begin by noting that since there is a unique level of contributions, \( D \), for each membership \( M \subseteq N \), Lemma 4 implies that we can write total contributions as a function \( D : \mathcal{P}(N) \rightarrow \mathbb{R}_+ \) such that

\[
(\forall M, M' \in \mathcal{P}(N)) \ M \subset M' \Rightarrow D(M) < D(M')
\]
where \( \mathcal{P}(N) \) denotes the powerset of \( N \). Now taking any subgame equilibrium with membership \( M \), define \( t^*_i(M) \), \( x^*_i(M) \), and \( a^*_i(M) \) to be the (subgame) equilibrium choices of leisure, private consumption and attendance for member \( i \). Note that these are functions also of \( w_i, D^*_i, \tau \), and \( \beta_i \) but we suppress these terms in the notation above. Thus, any member in a subgame equilibrium does not wish to leave the congregation if
\[
U^*_i(M) \equiv v(t^*_i(M), x^*_i(M)) + m(a^*_i(M), \mathcal{D}(M); \beta_i) \geq v(1, w_i).
\] (3)

Again in any subgame equilibrium with membership \( M \), consider now the incentives of any non-member, \( i \notin M \), of joining the congregation. A non-member must compare their reservation utility with the contribution equilibrium that would result if they joined the congregation. That is, they would join if
\[
v(t^*_i(M \cup \{i\}), x^*_i(M \cup \{i\})) + m(a^*_i(M \cup \{i\}), \mathcal{D}(M \cup \{i\}); \beta_i) \geq v(1, w_i).
\] (4)

In keeping with the notation introduced in (3), the left hand side of (4) can be written simply as \( U^*_i(M \cup \{i\}) \). If we let 1 denote membership and 0 denote non-membership, we can define a strategy that captures the incentive conditions in (3) and (4) by defining the function for all \( i \in N \):
\[
f_i(M) = \begin{cases} 1 & \text{if } U^*_i(M \cup \{i\}) \geq v(1, w_i) \\ 0 & \text{otherwise.} \end{cases}
\]

Note that this captures both the incentive conditions in (3) and (4) since if \( i \in M \), \( M \cup \{i\} = M \). To proceed, we first make note of the following useful characteristic of the functions \( f_i \).

**Lemma 5.** If we endow the powerset \( \mathcal{P}(N) \) with the partial order of set inclusion \( \subseteq \), then for all \( i \in N \), the function \( f_i : \mathcal{P}(N) \rightarrow \{0,1\} \) is monotonic (increasing) on \( \mathcal{P}(N) \).

The above lemma illustrates the complementarity in membership decisions among individuals in the presence of a tithing requirement. That is, individuals have greater incentive to join the congregation if they believe that the membership will be larger. One could argue that “evangelism”, or other efforts to increase awareness of a congregation within a community
are ways to promote this belief. This argument would suggest that congregations with institutionalized practices of giving would be more likely to engage in such activities. We exploit the complementarity demonstrated in Lemma 5 to establish the following proposition concerning the existence and structure of equilibria under tithing.

**Proposition 10.** For any $\tau \in [0, 1]$, the set of equilibria, $E(\tau)$, of the game satisfies the following:

1. $E(\tau)$ is non-empty.
2. There exists a largest and a smallest equilibrium in $E(\tau)$ (in terms of membership and total giving).
3. If $M$ and $M'$ are two equilibrium membership profiles, then $M \cup M'$ and $M \cap M'$ are also equilibrium membership profiles.

**Proof:** Define the function $F : \mathcal{P}(N) \to \mathcal{P}(N)$ by $F(M) = \{ i \in N : f_i(M) = 1 \}$. Note that since each function $f_i$ is increasing in $M$, so is the function $F$. Now since $\mathcal{P}(N)$ (endowed with the partial order of set inclusion, $\subseteq$) is a complete lattice, by Tarski’s Fixed Point Theorem, the set of fixed points of $F$ is also a complete lattice. Properties 1, 2 and 3 all follow from this fact.

It is worth noting that the words “largest” and “smallest” in the preceding proposition refer to the ordering of set inclusion, $\subseteq$. Thus the result that there is a “largest” (resp. “smallest”) equilibrium membership is stronger than a simple statement that there is an equilibrium with more (resp. less) members than any other equilibrium. Rather, the proposition above states that for any tithe $\tau$ there is a set of individuals that contains all equilibrium memberships, as well as a set of core individuals that is contained in every equilibrium membership. Using these results, we conclude this section by showing that the increase in total giving suggested by Proposition 9 can in fact always be achieved if the tithe $\tau$ represents a relatively small fraction of income.

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16Evangelism, in many ways, can be viewed simply as a type of advertising for the religious group in question. For an analysis of the coordinating role that advertising can play when decisions display complementarity see Rosborough (2008).
Proposition 11. **Consider an equilibrium with a set N of members and \( \tau = 0 \) in which total donations are \( D \) and the contributing set is \( C \). There exists a \( \tau' > 0 \) such that for any \( \tau \in (0, \tau'] \), total giving is always higher than in the equilibrium with \( \tau = 0 \).**

Note that this result suggests one potential motivating factor for the practice of tithing. A small tithing requirement can partially mitigate the under-provision problem typically associated with public goods funded through private donations. However, this increase in giving can also be coupled with a reduction in overall membership through the exclusion of those individuals who wish to attend, but are not willing to pay the amount of the tithe to do so.

5 Conclusion

Along with the main results above, two perhaps more subtle features of the model should be emphasized for empirical work. The first is that religious giving and attendance are both *choices* of individuals, and these choices can be affected by the decisions of others. This approach cautions that empirical studies using attendance as an exogenous variable to “explain” giving behavior are likely to have spurious results. Second, although there are measures of how strong individuals’ religious preferences are, there is clearly a large amount of unobserved heterogeneity among individuals, and these differences can affect giving and membership decisions. The result is that endogenous self-selection must on some level be a part of any data set on individuals in a congregation. One of the main goals of this paper is to provide a framework for empirical researchers to understand how this “sorting on unobservables” can affect the relationships between observable characteristics, such as income, and giving and attendance decisions.

The model presented here deals with a specific set of questions about giving, membership and attendance in religious congregations but there are many open questions for researchers in this area. Specifically, our model does not yet provide a functional role for overarching denominational bodies that can play an important role in affecting doctrine, training clergy,
and providing access to capital for many congregations. The results in our model suggest that the behavior of congregants can be dramatically affected by doctrine, thus endogenizing these decisions could provide useful insight into how denominations work. Analyzing the role of denominations in the context of a religious “market” with congregations of different denominational affiliations is the subject of our future work in this area.

Although the study of religious organizations from a rational choice perspective has become increasingly popular in other disciplines such as sociology, the economics literature on this subject remains relatively small. This paper contributes to the growth of this literature by applying many of the tools familiar in the economics of charitable giving to the unique set of incentives and constraints faced by individuals in religious congregations. Given their continuing importance in the charitable sector, a large amount of empirical work has been carried out on religious organizations and it is hoped that this paper can play a part in explaining some of the existing findings as well as in providing some new directions for researchers.

6 References


7 Appendix

**Proof of Proposition 1:** Define the sets $Z_i = [0, w_i]$ and $Z = \prod_{i=1}^{N} Z_i$. Then the functions $\delta_i : Z \to Z_i$ together form a single valued continuous function $\mathcal{F} : Z \to Z$. Since $Z$ is a compact, convex subset of $R^N$, by Brouwer’s Fixed Point Theorem a fixed point must exist. This fixed point is a Nash equilibrium vector of donations $\langle d_i^*, \ldots, d_N^* \rangle$.

To see that this defines a Nash equilibrium consider any individual $i$. Given $w_i$ and

$$D_{-i}^* = \sum_{j \neq i} d_j^*$$

$d_i^*$ is by definition a best response since it is constructed (via $\delta_i$) from the optimal choice $D_i^* = D_i(w_i + D_{-i}^*)$. Now, if $d_i^* > 0$ individual $i$ also chooses

$$x_i^* = x_i(w_i + D_{-i}^*), \ t_i^* = t_i(w_i + D_{-i}^*), \text{ and } a_i^* = a_i(w_i + D_{-i}^*)$$

Note that these choices are privately optimal and do not affect $D_{-i}^*$. In the case where $d_i^* = 0$, the individual sets $x_i^* = w_i$ and $(t_i^*, a_i^*)$ solve the reduced problem:

$$\max_{\{t_i, a_i\}} U(x_i, D, t_i, a_i) = v(t_i, w_i) + m(a_i, D_{-i}^*)$$

s.t. $t_i + a_i = 1$

The proof that normality of $x$ and $D$ yields uniqueness follows from Bergstrom, Blume & Varian (1986), Theorem 3 and is thus omitted.

**Proof of Proposition 2:** This proof follows Bergstrom et al.(1986). Given the equilibrium level of donations $D$ we have

$$\forall i \quad D^*(w_i + D_{-i}^*) \leq D \quad \text{with equality if } d_i^* > 0 \quad (5)$$

Assumption A3 implies that $\partial D^*/\partial W_i \in (0, 1)$, hence $D^*$ is invertible. Denoting the inverse of $D^*$ by $\phi(\cdot)$ we may apply it to both sides of (5) to get:

$$w_i + D_i^* \leq \phi(D) \quad \text{again with equality if } d_i^* > 0$$

Now since $D_{-i}^* = D - d_i^*$, this equation becomes

$$w_i - d_i^* \leq \phi(D) - D$$
Thus letting $\hat{w} = \phi(D) - D$ we have

$$w_i - d_i^* \leq \hat{w} \quad \text{with equality if } d_i^* > 0$$

or

$$(\forall i) \ d_i^* = \begin{cases} w_i - \hat{w} & \text{if } w_i > \hat{w} \\ 0 & \text{if } w_i \leq \hat{w}. \end{cases}$$

\[\square\]

**Proof of Proposition 4:** From Proposition 2, all non-contributors must have $w_i \leq \hat{w}$ and $\tilde{d}_i = 0$. Since the utility function is strictly increasing in $x_i$, this means that all non-contributors choose $\tilde{x}_i = w_i$. Notice that for all $i \in C$, $d_i^* = w_i - \hat{w}$ and so $x_i^* = \hat{w}$. This implies that for all non-contributors

$$(\forall i \notin C) \ \tilde{x}_i = w_i \leq \hat{w} = x^*$$

Thus given $w_i$ and $D$, non-contributors choose leisure and attendance to solve:

$$\max_{\{t_i, a_i\}} U(t_i, w_i, a_i, D) = v(t_i, w_i) + m(a_i, D)$$

s.t. $t_i + a_i \leq 1$ \hspace{1cm} (6)

An interior optimum for (6) is characterized by an attendance level $\tilde{a}$ such that:

$$v_t(1 - \tilde{a}_i, w_i) = m_a(\tilde{a}_i, D)$$

Notice that if $w_i < \hat{w}$ and the individual chose $(\tilde{t}_i, \tilde{a}_i) = (t^*, a^*)$ we would have:

$$v_t(t^*, w_i) < m_a(a^*, D)$$

since $v_{tx} > 0$ and $m_{aD} > 0$. Now since $v_{tt} < 0$, $m_{aa} < 0$ and $v_t \to \infty$ as $t \to 0$ it must be that

$$(\exists \tilde{a} \in (a^*, 1)) \ v_t(1 - \tilde{a}_i, w_i) = m_a(\tilde{a}_i, D)$$

So for non-contributors, $\tilde{a} \geq a^*$ and accordingly $\tilde{t} \leq t^*$ with strict inequalities if $w_i < \hat{w}$. Moreover, given $D$ we can differentiate the first order condition for non-contributors to find

$$\frac{d\tilde{a}}{dw_i} = \frac{v_{tx}}{v_{tt} + m_{aa}} < 0. \hspace{1cm} \square$$
Proof of Proposition 5: Since the conditions of Proposition 1 are satisfied, there is a unique equilibrium level of donations $D$ with a unique set of contributors $C$ for the congregation. Consider then the existence of a “marginal contributor” with $(\beta_i, w_i)$ such that

$$D_i^*(w_i + D_{-i}^*, \beta_i) = D_{-i}^*$$

(7)

Since the first order conditions of the problem in (1) would hold with equality for this individual, we can differentiate them with respect to $\beta_i$ to find

$$\text{sign} \left( \frac{\partial D_i^*}{\partial \beta_i} \right) = \text{sign} \left( m_{a}\beta v_{tx} - v_{tt} m_{D\beta} - m_{aD} m_{a\beta} \right)$$

where the expression on the right hand side above is positive from assumptions A1 and A2. Now given assumption A3, we apply the Implicit Function Theorem to (7) to find that the locus of potential types that could be “marginal contributors” in the $(\beta, w)$ plane is characterized by

$$\frac{dw_i}{d\beta_i} \bigg|_{D_i^* = D_{-i}^*} = -\frac{\partial D_i^*/\partial \beta_i}{\partial D_i^*/\partial w_i} < 0$$

\[ \square \]

Proof of Proposition 6: Let $\lambda$ and $\mu$ be the multipliers on the income and time constraints, respectively, for a positive contributor $i$. The equilibrium choices of $(t_i, x_i, d_i, a_i)$ are determined by the solution to the first order conditions in problem (1). Substituting these demand functions back into the first order conditions yields the identities:

$$v_x(t_i^*(w_i + D_{-i}; \beta_i), x_i^*(w_i + D_{-i}; \beta_i)) - \lambda_i^*(w_i + D_{-i}; \beta_i) \equiv 0$$

$$m_D(a_i^*(w_i + D_{-i}; \beta_i), D_i^*(w_i + D_{-i}; \beta_i)) - \lambda_i^*(w_i + D_{-i}; \beta_i) \equiv 0$$

$$v_i(t_i^*(w_i + D_{-i}; \beta_i), x_i^*(w_i + D_{-i}; \beta_i)) - \mu_i^*(w_i + D_{-i}; \beta_i) \equiv 0$$

$$m_a(a_i^*(w_i + D_{-i}; \beta_i), D_i^*(w_i + D_{-i}; \beta_i)) - \mu_i^*(w_i + D_{-i}; \beta_i) \equiv 0$$

$$w_i + D_{-i} - x_i^*(w_i + D_{-i}; \beta_i) - D_i^*(w_i + D_{-i}; \beta_i) \equiv 0$$

$$1 - t_i^*(w_i + D_{-i}; \beta_i) - a_i^*(w_i + D_{-i}; \beta_i) \equiv 0$$

(8)
Differentiating this system with respect to $\beta_i$ and applying Cramer’s Rule, we find:

\[
\left( \frac{\partial d^*_i}{\partial \beta_i} \right) \frac{\partial D^*_i}{\partial \beta_i} = \frac{-m_{a\beta} v_{tx} - v_{tt} m_{D\beta} - m_{aa} m_{D\beta} + m_{aD} m_{D\beta}}{|H|} > 0
\]

\[
\frac{\partial a^*_i}{\partial \beta_i} = \frac{-m_{aD} v_{xx} + v_{tx} m_{D\beta} + m_{a\beta} m_{D\beta} - m_{aD} m_{DD}}{|H|} > 0
\]

where the denominator $|H|$ is signed positive by the strict quasi-concavity of $U$. That donations are increasing in income follows from the normality of $D$, but to see how attendance varies with income we can differentiate the system in (8) with respect to $w_i$ and solve for:

\[
\frac{\partial a^*_i}{\partial w_i} = \frac{v_{tx} m_{DD} - v_{xx} m_a}{|H|}
\]

which has ambiguous sign. Consider next the choices of a non-contributor $j$. Given the equilibrium level of donations $D$ and income $w_j$, the time choices ($\tilde{t}_j, \tilde{a}_j$) are the solution to

\[
\max_{\{t_j, a_j\}} U(t_j, w_j, a_j, D; \beta_j) = v(t_j, w_j) + m(a_j, D; \beta_j)
\]

s.t. $t_j + a_j \leq 1$

(9)

Applying an analogous procedure as above to the first order conditions for (9) yields:

\[
\frac{\partial \tilde{a}_j}{\partial \beta_j} = \frac{m_{a\beta}}{-v_{tt} - m_{aa}} > 0 \quad \text{and} \quad \frac{\partial \tilde{a}_j}{\partial w_j} = \frac{v_{tx}}{v_{tt} + m_{aa}} < 0
\]

\[\square\]

Proof of Proposition 7: Note first that assumption A2 guarantees that all contributors will choose a positive level of attendance, since the marginal value of donations would be zero if these individuals chose $a_i = 0$. However, we may consider the existence of a “marginal member” among the set of non-contributors. This individual would choose $\tilde{a}_i(w_i, D, \beta_i) = 0$, but this decision would be on the boundary where the following tangency condition applies:

\[
v_t(1, w_i) = m_a(0, D; \beta_i).
\]

Thus we may characterize the locus of types that are “marginal members” by

\[
\left. \frac{dw_i}{d\beta_i} \right|_{\tilde{a}_i=0} = -\frac{\partial \tilde{a}_i/\partial \beta_i}{\partial \tilde{a}_i/\partial w_i} > 0
\]

where the sign of the above expression follows from Proposition 6. \[\square\]
Proof of Lemma 1: If both types are contributors, we know

\[ D^*(w + D^*_i; \beta^k) = D \quad \text{or} \quad w_i - d_i^k = \phi(D; \beta^k) - D \]  

(10)

where \( \phi(\cdot; \beta) \) is defined as the inverse of \( D^*(\cdot; \beta) \) around its first argument. Define \( C^k(\alpha) \) as the equilibrium set of contributors of type \( k \) for doctrinal position \( \alpha \), and \( c_k \) as the number of individuals in this set. We may then sum (10) over all \( i \in C(\alpha) \) to find that the equilibrium level of donations must satisfy:

\[ \sum_{i \in C^0} w_i + \sum_{i \in C^1} w_i = c_0 \phi(D; 1 - \alpha) + c_1 \phi(D; \alpha) + (1 - c_0 - c_1)D. \]  

(11)

Now consider the contributing behaviour of type 0 individuals. First, we know that \( d_i^0 \) is uniquely determined for any \( D \) and \( \alpha \) by

\[ d_i^0 = w_i - \phi(D, 1 - \alpha) + D \equiv w_i - \check{w}^0(\alpha). \]

This means that

\[ (\forall i \in C^0) \frac{\partial d_i^0}{\partial \alpha} < 0 \iff \frac{\partial \hat{w}^0}{\partial \alpha} > 0. \]

To reduce clutter, define \( \phi^k \equiv \phi(D; \beta^k) \) so that we may write

\[ \frac{\partial \hat{w}^0}{\partial \alpha} = (\phi^0_D - 1) \frac{\partial D}{\partial \alpha} - \phi^0_\beta. \]  

(12)

\( \partial D / \partial \alpha \) is found by implicitly differentiating (11). Thus we see that (12) is positive if

\[ -(\phi^0_D - 1) \cdot \left( \frac{-c_0 \phi^1_\beta + c_1 \phi^0_\beta}{c_0 \phi^1_D + c_1 \phi^0_D + (1 - c_0 - c_1)} \right) > \phi^0_\beta. \]

Since \( \phi_D > 1 \), the denominator of the left hand side is strictly positive so we may rewrite this expression as

\[ -(\phi^0_D - 1)(-c_0 \phi^1_\beta + c_1 \phi^0_\beta) > (c_0 \phi^0_D + c_1 \phi^1_D + (1 - c_0 - c_1))\phi^0_\beta. \]

Expanding and simplifying we see that this inequality holds if

\[ \phi^0_\beta + c_1(\phi^0_D - 1)\phi^1_\beta + c_1(\phi^1_D - 1)\phi^0_\beta < 0 \]

which must hold since \( \phi^k_D > 1 \) and \( \phi^k_\beta < 0 \) for \( k \in \{0, 1\} \). Note that a symmetric proof exists for type 1 individuals demonstrating that \( \partial \hat{w}^1 / \partial \alpha < 0 \), and thus \( (\forall i \in C^1) \partial d_i^1 / \partial \alpha > 0 \). \( \square \)
Proof of Lemma 3: To prove this result we make use of the following facts.

Fact 1: When $\alpha = 1/2$, $C^k$ is non-empty for $k \in \{0, 1\}$.

To see this, note that by assumption A5, there does not exist an $\alpha \in [0, 1]$ such that for $k \in \{0, 1\}$, $d_i^k(\alpha) = 0$ for all $i$. Thus, it must be the case for at least one of the types $k$ that $(\exists i) \ d_i^k > 0$. This in turn implies that

$$(\forall \alpha) \ (\exists i) \ w_i^k > \hat{w}^k(\alpha)$$

for at least one of the types, $k \in \{0, 1\}$. Now, by symmetry $\hat{w}^0(1/2) = \hat{w}^1(1/2)$. So when $\alpha = 1/2$, (13) implies that $C^k$ is non-empty for $k \in \{0, 1\}$ since the distribution of income levels is the same for both types.

Fact 2: When $\alpha = 1$, $\hat{w}^0(1) > w_i$ for all type 0 individuals.

This fact can be established by supposing instead that $(\exists i \in N^0)$ with $w_i \leq \hat{w}^0(1)$. This would imply that individual $i$ is a contributor and thus $i$’s decisions would satisfy the tangency condition

$$m_D(a_i^*, D; \beta_i) = v_x(t_i^*, x_i^*)$$

Now, assumption A4 implies that $a_i^* = 0$ for all type 0’s when $\alpha = 1$. But then assumption A2 also implies

$$m_D(0, D; \beta_i) = 0 < v_x(1, x_i^*)$$

A contradiction which establishes Fact 2.

Now, if we define $\overline{w}^0$ to be the highest income level among type 0 individuals, Fact 1 implies that $\overline{w}^0 > \hat{w}^0(1/2)$ and Fact 2 implies that $\overline{w}^0 < \hat{w}^0(1)$. Thus, the continuity of $\hat{w}^k$ established in Lemma 2 guarantees that there must exists an $\overline{\alpha} \in (1/2, 1)$ such that $\overline{w}^0 = \hat{w}^0(\overline{\alpha})$. Moreover, the monotonicity of $\hat{w}^k$ established in Lemma 1 guarantees that for all $\alpha' > \overline{\alpha}$ that $\hat{w}^0 > w_i$ for all $i \in N^0$, which implies that $C^0(\alpha')$ is empty. The proof that there exists an $\overline{\alpha}$ below which $C^1$ is empty is symmetric and is thus omitted.

Proof of Proposition 8: 1. Note that when $\alpha = 1/2$, $\beta_i^k = 1/2$ for all $i$. Thus the results of propositions 4 and 5 in the model with identical preferences apply. Namely, all contributors
choose \( a^*_i = a^* > 0 \) and all non-contributors choose \( \hat{a}_i > a^* \). Thus all individuals choose some positive level of attendance when \( \alpha = 1/2 \) and so membership size is \( 2n \).

When \( \alpha \in \{0, 1\} \), all individuals of type \( |\alpha - 1| \) choose \( a_i = 0 \) since assumption A4 implies \( v_i(1, w_i) > m_a(0, D; 0) \). Hence membership size when \( \alpha \in \{0, 1\} \) is \( n \).

2. Using 1, average giving when \( \alpha = 1/2 \) is

\[
AG(1/2) = \left( \frac{1}{2n} \right) \sum_{i \in C} d^*_i = \left( \frac{1}{2n} \right) \left[ \sum_{i \in C^0(1/2)} (w_i - \hat{w}^0) + \sum_{i \in C^1(1/2)} (w_i - \hat{w}^1) \right].
\]

But since \( \hat{w}^0(1/2) = \hat{w}^1(1/2) \equiv \hat{w}_{1/2} \), and the income distribution is the same for each type, we have

\[
AG(1/2) = \left( \frac{1}{2n} \right) \left[ 2 \sum_{i \in C^1(1/2)} (w_i - \hat{w}_{1/2}) \right] = \left( \frac{1}{n} \right) \left[ \sum_{i \in C^1(1/2)} (w_i - \hat{w}_{1/2}) \right]
\]

Now, when \( \alpha = 1 \), all type 0’s choose \( (t_i, x_i, a_i, d_i) = (1, w_i, 0, 0) \) and membership size is \( n \). So average giving is

\[
AG(1) = \left( \frac{1}{n} \right) \left[ \sum_{i \in C^1(1)} (w_i - \hat{w}^1(1)) \right]
\]

But notice that \( \hat{w}^1(1) < \hat{w}_{1/2} \) by Lemma 1, which in turn implies that \( C^1(1/2) \subseteq C^1(1) \) since the contributing set is determined by this threshold. Thus we may rewrite the above equation as

\[
AG(1) = \left( \frac{1}{n} \right) \left[ \sum_{i \in C^1(1/2)} (w_i - \hat{w}^1(1)) + \sum_{i \in C^1(1) \sim C^1(1/2)} (w_i - \hat{w}^1(1)) \right] \\
\geq \left( \frac{1}{n} \right) \left[ \sum_{i \in C^1(1/2)} (w_i - \hat{w}^1(1)) \right] \\
> \left( \frac{1}{n} \right) \left[ \sum_{i \in C^1(1/2)} (w_i - \hat{w}_{1/2}) \right] = AG(1/2)
\]

3. Define \( c_k(\alpha) = \#\{i \in C^k(\alpha)\} \). Then when \( \alpha = 1/2 \), the average income of contributors is

\[
AIC(1/2) = \left( \frac{1}{c_0(1/2) + c_1(1/2)} \right) \left[ \sum_{i \in C^0(1/2)} w_i + \sum_{i \in C^1(1/2)} w_i \right]
\]

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Again, since $\hat{w}^0(\frac{1}{2}) = \hat{w}^1(\frac{1}{2}) \equiv \hat{w}_2$, this expression is equal to

$$AIC(1/2) = \left(\frac{1}{2c_1(1/2)}\right) \left[ 2 \sum_{i \in C^1(1/2)} w_i \right] = \frac{\sum_{i \in C^1(1/2)} w_i}{c_1(1/2)}$$

Now Lemma 1 implies $\hat{w}^1(1) < \hat{w}_{\frac{1}{2}}$ so any individual who contributes when $\alpha = 1$ who contributed nothing when $\alpha = 1/2$ must have $w_i \in (\hat{w}^1(1), \hat{w}_{\frac{1}{2}})$. This implies that the average income among individuals in the set $C^1(1) \sim C^1(1/2)$ must be lower than average income among individuals in the set $C^1(1/2)$, or

$$AIC(1/2) = \frac{\sum_{i \in C^1(1/2)} w_i}{c_1(1/2)} > \frac{\sum_{i \in C^1(1) \sim C^1(1/2)} w_i}{c_1(1) - c_1(1/2)} \quad (14)$$

But notice that the average income of contributors when $\alpha = 1$ is

$$AIC(1) = \left(\frac{1}{c_1(1)}\right) \left[ \sum_{i \in C^1(1)} w_i \right] = \frac{\sum_{i \in C^1(1/2)} w_i + \sum_{i \in C^1(1) \sim C^1(1/2)} w_i}{c_1(1)}$$

$$= \frac{c_1(1/2) \left( \frac{\sum_{i \in C^1(1/2)} w_i}{c_1(1/2)} \right) + (c_1(1) - c_1(1/2)) \left( \frac{\sum_{i \in C^1(1) \sim C^1(1/2)} w_i}{c_1(1) - c_1(1/2)} \right)}{c_1(1)}$$

Notice that, from (14), this expression is simply a weighted average of $AIC(1/2)$ and a term that is smaller than $AIC(1/2)$. So $AIC(1) < AIC(1/2)$.

4. As noted above, when $\alpha = 1/2$, average attendance among contributors is $a^*(1/2)$. Attendance for any non-contributor, $\hat{a}_i$, is the solution to $v_i(1 - \hat{a}_i, w_i) = m_a(\hat{a}_i, D; 1/2)$. Thus, average attendance when $\alpha = 1/2$ is

$$AA(1/2) = \frac{[c_0(1/2) + c_1(1/2)]a^*(1/2) + \sum_{i \in N^0 \sim C^0(1/2)} \hat{a}_i(1/2) + \sum_{i \in N^1 \sim C^1(1/2)} \hat{a}_i(1/2)}{2n}$$

Again, since $\hat{w}^0(\frac{1}{2}) = \hat{w}^1(\frac{1}{2}) \equiv \hat{w}_2$, and the income distribution is the same for each type, we can rewrite this expression as

$$AA(1/2) = \frac{c_1(1/2)a^*(1/2) + \sum_{i \in N^1 \sim C^1(1/2)} \hat{a}_i(1/2)}{n}$$
which is simply the average attendance for all type 1 individuals when $\alpha = 1/2$. Thus average attendance will be higher when $\alpha = 1$ if we can show that attendance for all type 1 individuals is higher when $\alpha = 1$. To show this consider the following three possible groups of type 1 individuals.

a) Type 1 individuals with $w_i \geq \hat{w}_1$. These individuals contribute both when $\alpha = 1/2$ and when $\alpha = 1$. Moreover, Lemma 1 implies that $d_i(1) > d_i(1/2)$, which implies also that $x_i(1) < x_i(1/2)$ for these individuals. But under the assumption that $D(1/2) = D(1) = D$, this implies for these individuals

$$v_t(1 - a_i(1/2), x_i(1)) < m_a(a_i(1/2), D; 1)$$

since $v_{tx} > 0$ and $m_{a\beta} \geq 0$. Thus for this subset of type 1’s, it must be true that when $\alpha = 1$, $a_i(1) > a_i(1/2)$.

b) Type 1 individuals with $w_i \leq \hat{w}^1(1)$. These individuals do not contribute either when $\alpha = 1/2$ or when $\alpha = 1$. In this case, we can see for these types that

$$v_t(1 - a_i(1/2), w_i) \leq m_a(a_i(1/2), D; 1)$$

since $m_{a\beta} \geq 0$. So for this subset of type 1’s, it must be true that when $\alpha = 1$, $a_i(1) \geq a_i(1/2)$.

c) Type 1 individuals with $w_i \in (\hat{w}^1(1), \hat{w}_1)$. These individuals do not contribute when $\alpha = 1/2$ but contribute some positive amount when $\alpha = 1$. In this case $x_i(1) < x_i(1/2) = w_i$ so

$$v_t(1 - a_i(1/2), x_i(1)) < m_a(a_i(1/2), D; 1)$$

since $v_{tx} > 0$ and $m_{a\beta} \geq 0$. Thus for this final subset of type 1’s, it must also be true that when $\alpha = 1$, $a_i(1) > a_i(1/2)$.

\[\square\]

**Proof of Proposition 9:** We make use of the following fact that holds for any contribution equilibrium with total donations $D$ and contributing set $C$. This fact is proven in Bergstrom, Blume & Varian (1986). We include the proof here only for completeness.
FACT: There exists a real valued function $F(D,C)$, differentiable and strictly increasing in $D$, such that in a Nash equilibrium:

$$F(D,C) = \sum_{i \in C} w_i$$

To see this, note that from assumption A3, $D_i^*(\cdot)$ is a strictly increasing function and thus is invertible. Note that for all $i \in C$ we have $D_i^* = D$. Thus letting $\phi_i = D_i^{-1}(\cdot)$ we have:

$$\phi_i(D) = w_i + D_{-i} \quad \text{for all } i \in C.$$ 

Summing these equations we find

$$\sum_{i \in C} \phi_i(D) + (1-c)D = \sum_{i \in C} w_i$$

where $c$ is the number of individuals in the set $C$. Now define $F(D,C) \equiv \sum_{i \in C} \phi_i(D) + (1-c)D$ for any amount $D$ and set of individuals $C$. Then

$$F(D,C) = \sum_{i \in C} w_i.$$ 

Note that since $\phi_i' > 1$

$$\frac{\partial F(D,C)}{\partial D} = \sum_{i \in C} \phi_i'(D) + (1-c) > c + (1-c) = 1 > 0$$

This establishes the above fact.

To prove the proposition, take the equilibrium with tithing and for any set of individuals, $S$, let $W^S = \sum_{i \in S} w_i$. Then we know that for all potential members $i$:

$$D_i^* ((1-\tau)w_i + G_{-i} + \tau W^M) \leq D'$$

In particular this inequality must hold for all $i \in C$. Applying $\phi_i(\cdot) = D_i^{-1}(\cdot)$ to both sides gives

$$(1-\tau)w_i + G_{-i} + \tau W^M \leq \phi_i(D')$$

Note that $G_{-i} = G - g_i$ so we may rewrite this inequality as

$$(1-\tau)w_i + G - g_i + \tau W^M \leq \phi_i(D')$$
Summing these inequalities for all $i \in \mathcal{C}$ we have

$$(1 - \tau)W^C + cG - \sum_{i \in \mathcal{C}} g_i + c\tau W^M \leq \sum_{i \in \mathcal{C}} \phi_i(D').$$

where again $c$ is the number of individuals in the set $\mathcal{C}$. Rearranging terms gives

$$W^C + \tau (W^M - W^C) \leq \sum_{i \in \mathcal{C}} \phi_i(D') + \sum_{i \in \mathcal{C}} g_i + (1 - c)\tau W^M.$$ 

Now define $\mathcal{A}$ to be the set of individuals that give over and above the tithe. Then by definition $g_i > 0$ only if $i \in \mathcal{A}$, so it must be true that $\sum_{i \in \mathcal{C}} g_i \leq \sum_{i \in \mathcal{A}} g_i = G$. Thus the following inequality must hold:

$$W^C + \tau (W^M - W^C) \leq \sum_{i \in \mathcal{C}} \phi_i(D') + (1 - c) (G + \tau W^M).$$

Notice that $G + \tau W^M = D'$ so by definition of the function $F$, we may rewrite this as

$$W^C + \tau (W^M - W^C) \leq F(D', \mathcal{C}).$$

But $W^C = \sum_{i \in \mathcal{C}} w_i = F(D, \mathcal{C})$ from Lemma 1, so

$$F(D', \mathcal{C}) - F(D, \mathcal{C}) \geq \tau (W^M - W^C).$$

Notice that if $\mathcal{C} \subsetneq M$, the right hand side is strictly greater than zero, and since $F(D, \mathcal{C})$ is strictly increasing in $D$ we must have $D' > D$.

**Proof of Lemma 4:** Define $E_j$ as the equilibrium with membership $M_j$, for $j \in \{0, 1\}$. There are two cases to consider. The first is the case where all members in $E_0$ contribute exactly $d_i = \tau w_i$. In this case we must have:

$$D^0 = \tau \sum_{i \in M_0} w_i < \tau \sum_{i \in M_1} w_i \leq \tau \sum_{i \in M_1} w_i + \sum_{i \in M_1} g_i = D^1$$

The second case is where some individuals in $E_0$ give more than the tithe. Call this set of individuals $\mathcal{A}_0$. As before, define $W^S \equiv \sum_{i \in S} w_i$ for any set $S$. Notice that for all $i \in \mathcal{A}_0$ we have

$$D_i^* \left((1 - \tau)w_i + G_{-i}^0 + \tau W_{M_0}\right) = D^0$$
Applying $\phi_i(\cdot)$ to both sides and summing over all $i \in A_0$ yields

$$
(1 - \tau)W^{A_0} + (a_0 - 1)G^0 + \tau a_0 W^{M_0} = \sum_{i \in A_0} \phi_i(D^0).
$$

Rearranging terms we have

$$
W^{A_0} + \tau (W^{M_0} - W^{A_0}) = \sum_{i \in A_0} \phi_i(D^0) + (1 - a_0)D^0. \tag{15}
$$

Now, the right hand side of (15) is the function $F(D^0, A_0)$ defined in the proof of Proposition 9. We will use equation (15) to establish the result, and to do this we consider now the subgame equilibrium $E_1$ with membership $M_1$. Now, in this equilibrium, it must be true that for all $i \in M_1$

$$
D_i^* \left((1 - \tau)w_i + G_i^1 - \tau W^{M_1}\right) \leq D^1.
$$

If we again apply $\phi_i(\cdot)$ to both sides and sum over all $i \in A_0$ we have

$$
(1 - \tau)W^{A_0} + a_0 G^1 - \sum_{i \in A_0} g_i^1 + \tau a_0 W^{M_1} \leq \sum_{i \in A_0} \phi_i(D^1).
$$

Rearranging terms we have

$$
W^{A_0} + \tau (W^{M_1} - W^{A_0}) \leq \sum_{i \in A_0} \phi_i(D^1) - a_0 G^1 + \sum_{i \in A_0} g_i^1 + (1 - a_0)\tau W^{M_1}.
$$

Now since $g_i^1 > 0$ only if $i \in A_1$ it must be true that $\sum_{i \in A_0} g_i^1 \leq \sum_{i \in A_1} g_i^1 = G^1$. Thus the following inequality must also hold

$$
W^{A_0} + \tau (W^{M_1} - W^{A_0}) \leq \sum_{i \in A_0} \phi_i(D^0) + (1 - a_0) \left(G^1 + \tau W^{M_1}\right).
$$

or

$$
W^{A_0} + \tau (W^{M_1} - W^{A_0}) \leq F(D^1, A_0).
$$

Now, subtracting (2) from both sides of the above inequality gives us

$$
\tau \left( W^{M_1} - W^{M_0} \right) \leq F(D^1, A_0) - F(D^0, A_0).
$$

Note that since $M_0 \subset M_1$, the left hand side above is strictly greater than zero. Thus, given that $F(D, A)$ is strictly increasing in $D$, it must be the case that $D^0 < D^1$. \hfill \Box
Proof of Lemma 5: It suffices to show that for all $i \in N$, that if

$$f_i(M) = 1 \quad \text{then} \quad (\forall M' \in \mathcal{P}(N)) \quad M' \supseteq M \Rightarrow f_i(M') = 1.$$ 

To see this, first note that for all $i \in N$, if $M \subseteq M'$ then $M \cup \{i\} \subseteq M' \cup \{i\}$. Thus, by Lemma 1 we know that for all $i \in N$

$$M \subseteq M' \Rightarrow \mathcal{D}(M \cup \{i\}) \leq \mathcal{D}(M' \cup \{i\}). \quad (16)$$

To prove the main result, we must first establish that for all $i$,

$$x_i^*(M \cup \{i\}) \leq x_i^*(M' \cup \{i\}). \quad (17)$$

This means for any $i$, that equilibrium private consumption cannot be reduced when more members are added to the congregation. To see that (17) must hold, suppose to the contrary that for some $i$, we have $x_i^*(M \cup \{i\}) > x_i^*(M' \cup \{i\})$. Since $x_i^* = (1 - \tau)w_i - g_i^*$ it must then be the case that

$$g_i^*(M' \cup \{i\}) > g_i^*(M \cup \{i\}) \geq 0. \quad (18)$$

This implies that individual $i$ must give strictly more than the tithing requirement when membership is $M' \cup \{i\}$. If we define tithes and gifts for all individuals other than $i$ as

$$D_i^M = \sum_{j \in M \sim \{i\}} (\tau w_j + g_j)$$

then (18) implies:

$$D_i^*(w_i + D_i^M) = \mathcal{D}(M' \cup \{i\}) \quad \text{and} \quad D_i^*(w_i + D_i^M) \leq \mathcal{D}(M \cup \{i\})$$

Now since $D_i^*$ is increasing in $D_{-i}$, (16) implies that $D_i^{M'} \geq D_i^M$. However (18) also implies

$$D_i^*(w_i + D_i^{M'}) - D_i^M = g_i^*(M' \cup \{i\}) > g_i^*(M \cup \{i\}) \geq D_i^*(w_i + D_i^M) - D_i^M$$

but since $\partial D_i^*/\partial D_{-i} < 1$ the above inequality implies $D_i^{M'} < D_i^M$, a contradiction. Thus (17) must hold for all $i$.

We can now prove the main result by noting that $f_i(M) = 1$ implies

$$v(1, w_i) \leq v(t_i^*(M \cup \{i\}), x_i^*(M \cup \{i\})) + m(a_i^*(M \cup \{i\}), \mathcal{D}(M \cup \{i\}); \beta_i)$$

but since the right hand side of the above inequality is increasing in $x_i^*$ and $\mathcal{D}$, (16) and (17) imply that we must have, for any $M' \supseteq M$:

$$v(1, w_i) \leq v(t_i^*(M' \cup \{i\}), x_i^*(M' \cup \{i\})) + m(a_i^*(M' \cup \{i\}), \mathcal{D}(M' \cup \{i\}); \beta_i)$$

$$\leq v(t_i^*(M' \cup \{i\}), x_i^*(M' \cup \{i\})) + m(a_i^*(M' \cup \{i\}), \mathcal{D}(M' \cup \{i\}); \beta_i)$$
which implies that $f_i(M') = 1$. □

**Proof of Proposition 11:** Consider the original equilibrium with $\tau = 0$ and contributing set $C$. Define
\[
\tau_C = \min \left\{ \frac{D^*_i(w_i; \beta_i)}{w_i} \mid i \in C \right\}
\]
as the minimum proportion of income that any original contributor would give if all other individuals gave nothing. Next for non-contributors define
\[
\tau_N = \max \left\{ \frac{D^*_i(w_i; \beta_i)}{w_i} \mid i \notin C \right\}.
\]
Then set $\tau' = \min\{\tau_N, \tau_C\}$. Consider first the case where $M = \phi$. By construction, $f_i(\phi) = 1$ for all $i \in C$ and $(\exists i \in \sim C)$ such that $f_i(\phi) = 1$. Thus we can define
\[
M_1(\tau') = \{i \in N \mid f_i(\phi) = 1, \tau = \tau'\}.
\]
Notice that by lemma 5, $f_i(\phi) = 1 \Rightarrow f_i(M_1(\tau') \cup \{i\}) = 1$. So all these individuals will join and, by Proposition 10, they must be part of every equilibrium membership profile $M^*(\tau')$.

Following the above reasoning, note that this is also true for any $\tau \in (0, \tau')$. The desired result is then a consequence of Proposition 9 and the fact that any equilibrium $M^*(\tau')$ must satisfy $M^*(\tau') \supseteq M_1(\tau') \supseteq C$. □